Consistent Tax on Non-Renewable Resources Revisited

Florian Habermacher*

February 16, 2013

Preliminary draft, do not cite without permission by the author

Abstract

In his seminal contribution on optimal and consistent taxes on non-renewable resources, Karp (1984) finds that a single buyer restricted to a consistent (uncommitted) tax path, and facing a competitive supply, will not impose a tax of zero when the competitive outcome implies a zero resource rent for the last unit of stock. He also finds that equilibrium taxes must be zero with two non-committing Nash-competing buyers and a competitive supply. Both claims seem wrong, the optimal consistent taxes are in general positive. We identify the mistake in Karp's analysis, and proof that the zero-tax cannot be the optimal uncommitted policy, and provide expressions for the optimal consistent tax, which is positive for any finite number of buyers. We illustrate our theoretical analysis with a dynamic numerical example for exhaustible oil. A convergence result for an ever smaller commitment period confirms the inconsistency of Karp's result, and we illustrate the optimal uncommitted tax path according to our result which is, as expected, situated between the optimal committed tax and the abscissa. We also show how our results extend to the case with externalities and leakage.

JEL-Codes: Q41, H21, Q54, H23.

Keywords: time consistent optimal tax, international trade of exhaustible resource, subgame-perfectness, energy taxation, unilateral climate policy, dynamic carbon leakage.

Swiss National Science Foundation Supported by the SNSF (Swiss National Science Foundation).

*Swiss Institute for International Economics and Applied Economic Research (SIAW), University of St. Gallen, Bodanstrasse 8, CH-9000 St. Gallen, Switzerland. florian.habermacher@unsig.ch. Visiting researcher at Oxford Centre for the Analysis of Resource Rich Economics (OxCarre), Department of Economics, University of Oxford, UK.

Introduction

In his seminal contribution, Karp (1984) considers time-consistent optimal buyer-tariffs in a model with competitive supply of an exhaustible resource with stock dependent extraction costs. He asserts that (i) the optimal, uncommitted tax by a monopsonist buyer leaves the extraction path unaffected (i.e., the purely competitive extraction path without taxes obtains), and (ii) if there are two non-cooperative buyers (implying a Nash competition between the buyers and a Stackelberg-follower seller), their optimal uncommitted competitive taxes will be zero. As Karp mentions, the formulation of (i) has two implications: (a) if the rent on the final unit of stock is positive, the buyer can impose a tax to extract that rent, driving the sellers' final resource rent to zero, using a tax with a constant NPV level to leave the extraction path unaffected, and (b) if instead the rent on the final unit of stock is zero, the taxes must all be zero as well (Karp, p. 89). If valid, Karp's results would have obvious, important implications for the optimal fiscal (trade) policy of resource importing countries. They implied that, contrary to standard case of non-exhaustible goods offered at increasing prices, for exhaustible fuels, terms-of-trade cannot be improved through unilateral import taxes, if the region cannot commit to upholding this tax also in the future. The present analysis shows, however, that this view is mistaken, and instead the optimal buyer taxes are (nonmarginally) positive even in the absence of any commitment to future taxes.

Chapter 1 pinpoints a mistake in Karp's proof[1] for his Proposition 2 where he had generalized claims (i) and (ii).[2] The chapter also gives a tentative explanation for why Karp's assertions had found wide acceptance despite their erroneousness.

Chapter 2 considers optimal consistent taxes. The basic intuition for the non-zero optimal tax result used throughout the chapter is as follows. A buyer-tax-induced consumption reduction, sustained for an infinitesimal policy-period, has only a vanishingly small direct marginal deadweight loss on the buyer's instantaneous utility flow. It has, however, a first-order effect on the policy-period consumption rate, which in turn implies an increase of the stock at the end of that small period. As the NPV of the buyer's total post-policy utility increases with the initial post-policy resource stock (with a nonmarginal slope), there is a first order gain from resource savings, opposed only by the second order loss in the policy period itself. This is possible because, despite not changing the resource-gains of the seller in terms of their stock-dependent *current* values, the policy has, by delaying extraction, an effect on the relevant *present value* of the sum of all future seller gains, allowing a partial rent shift from the seller to the buyer, that appears to outweigh the total welfare loss due to the distorting tax.

[1] Appendix B in Karp, pp. 94.

[2] P. 87 in Karp.

The chapter explains why Karp's results can impossibly hold (section 2.1), as they lead to a logical contradiction. Section 2.2 provides an expression for the optimal consistent path of uncommitted taxes containing strictly positive rates as long as a non-marginal amount of resource stock is available.

Section 2.3 generalizes the result to the case of several buyers. Subsequently, section 2.4 explains how the results seem to extend directly to the case with pollution.

Chapter 3 illustrates and confirms the general results in a numerical model (calibrated for the worldwide market for exhaustible oil), notably confirming that even as the commitment duration approaches zero, the tax for the initial period does not approach zero, but instead converges towards a positive constant close to the value of the commited tax.

This work provides the conditions for the optimality of the consistent taxes in a raw format; they readily allow the calculation of the optimal time-consistent taxes notably in numerical models. They may be further developed to expressions based on more fundamental variables than the utility implied by the resource consumption and prices. However, they clearly show that the optimal consistent taxes are in general positive, reversing Karp's seminal result, requiring also additional works subsequent to Karp's to be reconsidered.

Throughout, the buyer, whose optimal consistent policy we study, is assumed to choose a domestic utility-maximizing tax path under the constraint that he cannot commit to future taxes, and subject to the Stackelberg reaction of competitive sellers.

1 Karp's analysis

The following considers Proposition 2 in Karp's paper, which claims that buyers of an exhaustible, competitively supplied resource cannot use taxes to increase their welfare, if they are restricted to time-consistent policies.

Consider thus a dynamic open-horizon framework with a resource owned by competitive suppliers and, for the moment, a monopsonistic buyer who can buy resource units only for direct consumption (that is, the buyer cannot store the resource once the suppliers have extracted it).

Whilst Karp formulates it in a somewhat generalized form,[3] for our case, Proposition 2 in Karp considers the case of the maximization of the buyer's welfare,

$$\begin{aligned} J_B &= \max_x \int_0^T (e^{-rt} (u(x) - c(z)x) - x\lambda \mathrm{d}t, \qquad (1) \\ &\text{s.t.} \quad \dot{\lambda} = e^{-rt} c'(z) x, \end{aligned}$$

[3] Mainly writing $J_B = J_B = \max_x \int_0^T (e^{-rt} g(x,z) - xh\lambda \mathrm{d}t$ instead of (1), with h a constant.

subject to usual regularity conditions for resource extraction and with $c(z)$ the stock depending marginal extraction costs with $c'(z) < 0$ and the stock z evolving according to $\dot{z} = -x$. The proposition states that in this case, the impossibility for the buyer to commit to future policies would imply that he acts as if maximizing

$$J_B \quad = \quad \max_x \int_0^T (e^{-rt} (u(x) - c(z)x) \, \mathrm{d}t,$$

i.e., as if he wouldn't have to pay any resource rent λ_t to the producer (cf. p. 87 and Eq. 7a on p. 79 in Karp, 1984).

To make his case, he assumes, for simplicity, that $z^* = 0$, i.e., all of the resource may be economically extracted. He requires the buyer to set (revise) policy whenever the cumulative consumption since the last revision equals $\bar{z}/n$, where $\bar{z}$ is the initial stock and $n \to \infty$. Times of revision are defined as t_i, $i = 0, 1, \ldots, n-1$, so $z(t_i) - z(t_{t+1}) = \bar{z}/n$, and $\delta(i) \equiv t_{i+1} - t_i$.

The $n$revision payoff is then (A.1 in Karp)

$$J_B(\bar{z}, t_0, n) = \sum_{i=0}^{n-1} \int_{t_i}^{t_{i+1}} (e^{-rt} (u(x) - c(z)x) - x\lambda \mathrm{d}t.$$

Using a sort of a present-to-current value variable transformation to define the time-invariant $\lambda^*(i \cdot \bar{z}/n)$ and $J_B^*(i \cdot \bar{z}/n)$ with $\lambda(t_{n-i}, i \cdot \bar{z}/n) = e^{-rt_{n-i}} \lambda^*(i \cdot \bar{z}/n)$ and $J_B(i \cdot \bar{z}/n, t_{n-i}, n) = e^{-rt_{n-i}} J^*(i \cdot \bar{z}/n)$, where instead of the $i \cdot \bar{z}/n$ we could also have written $z(t_{n-i})$.

He rightly finds that, for small $\delta(i)$ (i.e., as $n \to \infty$), we can write (A.3 in Karp)

$$J_B(\,) = \sum_{i=0}^{n} \left[e^{-rt_i} (u(x) - c(z)x(t_i)) - e^{-rt_i} x(t_i) \lambda^*(z(t_i)) \right] \delta(i) + o(\delta(i)), \tag{2}$$

and that the products $x(t_i)\delta(t_i)$ are given by the definition of the revision times, implying $x(t_i)\delta(t_i) = \bar{z}/n$, i.e., they cannot be influenced by the policy. Problematic is when he concludes that therefore "of the two terms in the summand [in (2)], only the first can be influenced by current controls", and that therefore, in the limit of (2) for $n \to \infty$ and $\delta(i) \to 0$,

$$J_B(\,) = \int_0^T e^{-rt} (u(x) - c(z)x) \, \mathrm{d}t - \int_0^T x\lambda \mathrm{d}t, \tag{3}$$

consistency would force the buyer to "act as if he were maximizing only the first integral [in (3)]", wherewith he concludes his proof. However, by influencing $\delta(i)$, the policy revision from time t_i, does affect the second term in the summand of (2) for all future 'periods' $j > i$ (and therewith the second integral in (3)), through the exponent in the discount factor e^{-rt_j}, where $t_j = t_0 + \delta(1) + \ldots + \delta(i) + \ldots + \delta(j-1)$. This impact on the second terms in the summand in (2) for all periods $j > i$ implies, in total, a change that does not vanish more rapidly with the smallness of $\delta(i)$ than the change in the first terms of the summand. The proof is therefore not valid. As the contradiction below, as well as our expressions for the non-zero optimal consistent tax show, Karp's Proposition 2 and his assertions about the consistent buyer taxes cannot be valid.

Correspondingly, also the "economics underlying the mathematics" is not as clear as Karp asserts in the Appendix after his proof; in his argument I cannot find any compelling, intuitive argument that would naturally imply the result postulated by Karp. Instead, the only point there is with respect to the multiplier λ being fixed, is that a buyer policy that influences the current consumption rate during an infinitesimally short period without affecting the future policy (as would be the case if the tax would anyways be set to zero in future), can change the current value of the multiplier λ – and that of all future periods – only by an infinitesimally small value, and that for the future periods the tiny change corresponds simply to a shift of the time path slightly into the future. But that does not mean that the whole λ path must be considered as fixed; after all, we consider a policy of an infinitesimally small duration; if that policy changes λ for the remainder of the whole time horizon (or at least for a substantial time), this can be enough to outweigh a decrease of the utility flow during the considered initial policy period, as will be shown below. That the buyer may have to lower his optimal tax path if he is restricted to time-subgame-perfect (i.e. consistent) policies, does not imply that he has to set the tax rates to zero.

It is speculation, but the trust in Karp's results may ultimately have relied on the valid recognition that the reduction of current consumption cannot truly influence the future stock-related current value shadow multipliers (λ^* in Karp) and basically only shifts the whole extraction and consumption paths slightly into the future. It was neglected that, the more directly relevant *present*-value producer rent (corresponding to the multipliers λ) are, however, affected; delaying the extraction shifts the stock-path into the future and lowers the corresponding present-value multipliers, implying that current reductions do affect the relevant rent-value paid to the producer. This comes not without current cost to the buyer. As we have shown, this cost is, however, for low enough (but above-marginal) current taxes, necessarily outweighed by the buyer's gain in the future (which can also simply be considered as a prolongation of the resource availability). The recognition of the pure-shift effect of current consumption choices on future consumption, and the tradeoff between the buyer's current cost from the tax and the corresponding future benefits are what below allows us to derive the valid expression of the optimal consistent policy.

2 Optimal consistent policy

For the uncommitted, time-consistent policy, there are mechanisms that in general make the solution extremely more complicated than in the case of standard, open-loop (committed) optimization. Consider the general case of resource extraction, assuming, e.g., a non-constant demand. Past policies must take into account today's rational policy choice, but per se, when considering today's rational policy itself, effects on past times are discarded, quasi by definition of the time-consistency. What complicates the matter is, that today's policy choices affect future policies, and, just because these future policies are again 'only' uncommitted ones, they cannot be considered as genuinely optimal from today's perspective and our today's effect on them can therefore not be ignored as we would have done in general (Envelope theorem) in a more standard problem. This can make the problem very difficult to deal with. When the problem exhibits a stationarity in the sense that future policies depend only on the stock and not on the time, the time-invariance of the problem can offer a way for relatively simple solution. In the standard resource extraction model this is the case when demand is stationary (as is the marginal extraction cost curve as a function of cumulative extractions, as is very typically assumed in literature, even though this assumption could obviously be questioned). In this case, and it is this one that is treated in Karp and here, we are spared from explicitly taking into account the effects of our current policy on all future policies, at least for the derivation of one form of the optimality characterizing the optimal policy.

As we know, the competitive solution maximizes total welfare. As we also know, with a committed policy, the buyer can artificially reduce his consumption, lower prices and therewith extract parts of the resource rent that else falls upon the seller. As we will see, to a certain extent, this will still be possible for the case of a time-consistent, uncommitted buyer, and even in the case where Karp thought to had ruled it out, when the last unit of (economically extractable) resource has a value of zero.[4] The policy may reduce overall welfare, but the shift of rent from the seller to the buyer can be larger than that the social loss (with the simple intuition for it being that for an infinitesimally short policy period, a domestic consumption tax is associated with the usual 'second order only' deadweight loss, but a first order effect in terms of the amount of fuels saved for the future).

To make it as clear as possible that Karp's result cannot hold, section (2.1) looks at the optimal instantaneous policy under the hypothetical assumption that the future policy will be one of always zero taxes. Whilst this does not yet give per se the optimal instantaneous uncommitted tax, the finding of a strictly positive optimal instantaneous tax for this case shows that Karp's solution cannot be an equilibrium: if he was right, then the future policy would indeed be zero taxes throughout (and notably so independent of today's

[4] As is the case, e.g., for a demand with a finite choke price (or, thus, when a constant price backstop is available) and an extraction cost curve that smoothly crosses this choke.

tax, as Karp's policy is stock-independent), wherewith it would indeed have been valid to consider the future tax to be zero anyways, and the therewith here found, positive optimal current tax for this case would have to be valid, contradicting Karp. In a second point we adapt the analytics to the relevant case where current and future taxes may be nonzero: sections (2.2) through (2.4) consider the case with a single buyer, the extension to multiple buyers, and the case with pollution.

2.1 Future taxes hypothetically set to zero (a contradiction for Karp's result)

Consider the competitive outcome as a starting point, indexed by *. Importantly, note that we will throughout be concerned only with the *current* value shadow value, called λ_t and the specific solution for the competitive outcome thus here λ_t^*; this is in contrast to Karp (Appendix B), who reserves λ^* specifically for the *current* value multiplier and denotes the *present* value counterpart as λ_t.

For p_t the seller price, we always have

$$p_t = c_t + \lambda_t \tag{4}$$

$$\lambda_t = \int_t^{\infty} e^{-\rho(s-t)} \dot{c}_s \mathrm{d}s, \tag{5}$$

and for a buyer's tax on the decentralized consumption, we have

$$u'(x_t) = p_t + \tau_t, \tag{6}$$

and in the competitive solution ($\tau_t^* = 0$),

$$u'(x_t^*) = p_t^*. \tag{7}$$

Instantaneous buyer utility is $U_t \equiv u(x_t) - p_t x_t$, with total NPV welfare being $W \equiv \int_0^{\infty} e^{-\rho t} U_t \mathrm{d}t$.

We assume that the usual regularity conditions apply.

For a normalized time $t = 0$, and an infinitely closely following period $\delta \to 0^+$, the regularity of the problem implies $lim_{t\to\delta} p_t = p_\delta$, $lim_{t\to\delta} c_t = c_\delta$, $lim_{t\to\delta} \lambda_t = \lambda_\delta$, and, at least in the competitive equilibrium, $lim_{t\to\delta} x_t^* = x_\delta^*$. Call the policy period $T1 = \{t \,|\, t \in [0, \delta]\}$, and the remainder of the horizon, $T2 = \{t \,|\, t > \delta\}$. Consider a policy yielding a constant consumption reductions by r during $T1$, $x_{T1} = x_{T1}^* - r$. The marginal reaction of U_t to that reduction r of x_t during $T1$ is $-u'(x_t) + p_t$ (thus zero for marginally low taxes, due to (6) and (7), as expected for marginal deadweight losses), and, as $\delta \to 0$, the marginal reaction of total NPV buyer utility for period $T1$ to a marginal change in the reduction r is approximated as[5]

$$\int_{T1} \underbrace{e^{-\rho t}}_{\to 1} \frac{\partial U_t}{\partial r} \mathrm{d}t = \delta \left[-u'(x^*_\delta - r) + p^*_\delta \right] + o(\delta). \tag{8}$$

The second effect of the marginal change of the policy r, is a time-shift of the $T2$ consumption and utility (etc.) paths into the future,[6] with the marginal of the time-shift s w.r.t. the policy variable validly approximated as the marginal of the $T1$ resource saving due to the policy, $\delta \cdot \frac{-\partial x}{\partial r} = \delta$, divided by the rate of consumption during the shift period approximated as x^*_δ, yielding $\frac{\partial s}{\partial r} = \frac{\delta}{x^*_\delta}$. The other variables during that 'wedge' period lasting for a duration s from δ on, are also validly approximated by their $(\cdot)^*_\delta$ values. The marginal reaction of instantaneous utility U_t to a s-period shift of the consumption path around t is given by $\frac{\partial U_t}{\partial s} = -s\dot{U}_t$. Therewith, the total marginal reaction of the $T2$ welfare to marginal policy changes r can be approximated as

$$\frac{\partial \int_\delta^\infty e^{-\rho t} U_t \mathrm{d}t}{\partial r} = -\frac{\delta}{x^*_\delta} \int_\delta^\infty e^{-\rho t} \dot{U}^*_t \mathrm{d}t. \tag{9}$$

The FOC for the optimal policy, which requires current costs, (8), and future benefits, (9), for marginal policy changes to offset, thus gives

$$u'(x^*_\delta - r) - p^*_\delta = -\frac{1}{x^*_\delta} \int_\delta^\infty e^{-\rho t} \dot{U}_t \mathrm{d}t.$$

With p_{T1} still validly approximated by p^*_δ and the decentralized consumption subject to taxes, (6), that gives the simple expression for the optimal tax,

$$\tau_0 = -\frac{1}{x^*_\delta} \int_\delta^\infty e^{-\rho t} \dot{U}_t \mathrm{d}t. \tag{10}$$

Because resource prices rise over time as depletion advances, we generally have $\dot{U}_t < 0$ and thus positive taxes, $\tau_t > 0$. This contradicts Karp.

[5]Note, (4) and (5) imply that p_{T1} is only marginally changed, with a vanishing relative effect during that short period itself.

[6]A more intuitive way to understand that the remainder of the analysis warrants a strictly positive tax is that $T2$ overall NPV utility is a function that smoothly and nonmarginally increases with the resource stock S_δ, as is generally found in non-renewable resource extraction models and shown to be the case in a simple setup in the Annex A.1. Given that the marginal deadweight loss from the tax during $T1$ is vanishingly low for $\tau_{T1} = 0$, this implies a strictly positive optimal value for the optimal tax rate.

2.2 Optimal uncommitted tax path

We are now looking for the equilibrium for the optimal committed policy path, following very closely the analysis from the previous section 2.1.[7]

First note that, as we assume that the usual regularity conditions apply, it is clear that the policy itself evolves smoothly over time, that is, we have finite derivatives for all variables on our solution path.

For a normalized time $t = 0$, and an infinitely closely following period $\delta \to 0^+$, the regularity of the problem thus implies $lim_{t\to\delta} p_t = p_\delta$ and $lim_{t\to\delta} x_t = x_\delta$. Call the policy period $T1$, $T1 = \{t \,|\, t \in [0, \delta]\}$, and the remainder of the horizon $T2$, $T2 = \{t \,|\, t > \delta\}$. The policy defines a constant $T1$ consumption rate X, $x^*_{T1} = X$ (indirectly through the tax; given that we have vanishingly small reactions and variations of the seller price, fixing either of them is equivalent). Consider a marginal change of the policy X. The marginal reactions of utility flows U_{T1} to that change are $u'(X) - p_{T1}$, and, as $\delta \to 0$, the marginal reaction of total NPV buyer utility for period $T1$ to a marginal change in X can be approximated as[8]

$$\int_{T1} \underbrace{e^{-\rho t}}_{\to 1} \frac{\partial U_t}{\partial X} \mathrm{d}t \quad = \quad \delta \left[u'(X) - p_\delta\right] + o(\delta). \tag{11}$$

Call S_t the stock of resources. Marginal changes of X during $T1$ will change the stock at the end of $T1$, S_δ at the rate $\frac{\partial S_\delta}{\partial X} = -\delta$. This is a marginal effect on the stock, and has a negligible effect on consumption rate and utility flow directly after phase $T1$: $\lim_{t\to\delta^+} \frac{\partial x_t}{\partial X} = 0 + o(X)$ and $\lim_{t\to\delta^+} \frac{\partial U_t}{\partial X} = 0 + o(X)$. The marginal increase in the stock S_δ can thus be considered as having two effects, in approximation: introducing an intermediate phase, right after δ, with a utility and consumption flow that corresponds to the original values at the beginning of $T2$, and a corresponding time-shift of the original equilibrium $T2$ path for all variables by the corresponding time into the future. The marginal reaction of instantaneous utility U_t to an marginal k-period shift of the consumption path around t is given by $\frac{\partial U_t}{\partial k} = -k\dot{U}_t$, and a variation in the stock, $\mathrm{d}S_\delta$ translates into a time-shift $\mathrm{d}k$ by $\frac{\mathrm{d}S_\delta}{x_{\delta+}}$. The effect of X on the phase $T2$ implies thus an overall $T2$ utility effect that can be approximated as

$$\frac{\partial \int_\delta^\infty e^{-\rho t} U_t \mathrm{d}t}{\partial X} = \frac{\delta}{x_\delta} \int_\delta^\infty e^{-\rho t} \dot{U}_t \mathrm{d}t. \tag{12}$$

The FOC for the optimal policy, which requires current costs, (11), and future benefits,

[7] Note that we will throughout be concerned only with the *current* value shadow value, called λ_t ; this is in contrast to Karp (1984, Appendix B), who reserves λ^* specifically for the *current* value multiplier and denotes the *present* value counterpart as λ_t.

[8] Note, (4) and (5) imply that p_{T1} is only marginally changed, with a vanishing relative effect during that short period itself.

(12), for marginal policy changes to offset, thus gives

$$u'(x_\delta - r) - p_\delta = -\frac{1}{x_\delta}\int_\delta^\infty e^{-\rho t}\dot{U}_t \mathrm{d}t.$$

With p_{T1} still validly approximated by p_δ, and with decentralized consumption subject to taxes, (6), this gives the simple expression

$$\tau_0 = -\frac{1}{x_\delta^*}\int_\delta^\infty e^{-\rho t}\dot{U}_t \mathrm{d}t. \tag{13}$$

Since, besides the general time-shift, future policies are not affected by today's choice, it is no surprise that this expression is identical to that derived above for the case of the absence of future taxes, (10). As the problem is time-invariant, (13) can be generalized for the other periods, and, as $\delta \to 0$ it can be simplified, to characterize the optimal, subgame-perfect tax path τ^* as follows:

$$\tau_t^* = -\frac{1}{x_t}\int_t^\infty e^{-\rho(v-t)}\dot{U}_v \mathrm{d}v. \tag{14}$$

There is no reason to expect this path to consist of zero taxes. First, as shown above, a path with zeroes everywhere leads to a contradictory result. Furthermore, the stationarity of the problem strongly suggests that even when future taxes are set subgame-perfectly optimal, they do not prevent the utility flows U_t from decreasing over time: the stationarity implies that taxes preventing all consumption during t cannot be optimal,[9] and positive consumption implies that the stock is decreasing over time, which in general leads to increasing equilibrium prices and decreasing optimal fuel consumption rates, implying strictly positive tax rates in (14).

2.3 With Active or Passive Competing Buyers

It is easy to extend (14) to the case competing buyers, showing that, contrary to Karp's assertion, also then, positive taxes obtain. Rather than going through all the analytics again, we just explain how the above for the optimal tax path of the monopsonist must be adapted. We start by considering a single, Nash competing buyer. First, note that imposing a tax sustained only during an infinitesimally small period $T1$ up to δ will lead to the same period $T1$ loss as in the case of the single buyer above, given in (11). This holds, as during $T1$ the pre-tax market price of the resource is still validly approximated as a constant unaffected by the $T1$ policy, which also implies that the foreign competitor's *concurrent* consumption is validly approximated as unaffected by that policy. The buyer-

[9] It would purely shift the extraction path into the future, which must lead to a loss given the positive time-discount rate.

profit in future time periods will, however, be shared between the two consumers, which, in the simplest case of perfectly symmetric demand functions (other cases are discussed in a moment and do not prevent the contradiction from arising), implies that the $T1$ gain from the previous section is reduced to half of (12). Obviously, this still warrants a strictly interior solution with a non-marginal, positive tax rate, now reduced by a factor $\frac{1}{2}$,

$$\tau_t^* = -\frac{1}{2x_t}\int_t^\infty e^{-\rho(v-t)}\dot{U}_v \mathrm{d}v. \tag{15}$$

Interestingly, this result generalizes to the case of non-symmetric buyers in a simple way. The factor 2 in (15) arises because the foreign consumption halves the marginal consumption path delay induced by a domestic $T1$ consumption reduction. The analytics make it clear that except for this effect on the delay time, the details of the form of the foreign consumption choices are not relevant for the optimal domestic, committed tax. Ultimately this implies that the optimal tax for a competitor in a Nash game always corresponds to (15), with the factor $\frac{1}{2}$ adapted to the competitor's share of the global period-t consumption, and does not explicitly depend on the form of the foreign competitor's demand function. The analytics above further extend to the case of more than two competitors. Furthermore, it is straightforward to see that even for a monopsonist with a passive fringe, or for several Nash competitors with a passive fringe, the result does not change.

We emphasize thus the following result.

Proposition A. The time-consistent unit tax of a buyer of a non-renewable resource, extracted with increasing extraction cost and competitively supplied by a Stackelberg follower, potentially facing competition by a finite number $n \in [0, \infty)$ of consistently Nash competing and/or passive[10] buyers, follows

$$\tau_t^* = -\frac{\xi_{i,t}}{x_t}\int_t^\infty e^{-\rho(v-t)}\dot{U}_{i,v} \mathrm{d}v,$$

with $\xi_{i,t}$ the time t consumption share of competitor i, $\xi_{i,t} \equiv x_{i,t}/x_t$.

2.4 Extension: Stock Pollutant

Consider an externality attached to the global consumption of the resource, with a consumption emission that is persistent and a damage cost convex in cumulative emissions. Normalizing, without loss of generality, the emission intensity of the resource to 1, and denoting cumulative consumption $X_t = \int_0^t x_v \mathrm{d}v$, $x_t = \Sigma_i x_{i,t}$, and its damage $D(X)$, we

[10] Passive means governed by undistorted decentralized consumption.

thus write the buyer's utility flow as

$$U_{i,t} = u(x_t) - p_t x_t - D(X_t). \tag{16}$$

His overall welfare is $W_i = \int_0^\infty U_{i,t} \mathrm{d}t$.

How does the presence of the externality D affect the optimal, time-consistent tax?

Given that D is purely a stock-dependent function,[11] the insight that a current consumption change affects the future basically by a simple time-shift still applies. We again consider the infinitesimally short policy period $T1 = \{t \,|\, t \in [0, \delta]\}$, with $\lim_{\delta \to 0^+}$, and the time afterwards, $T2 = \{t \,|\, \mathrm{t}{>}\delta\}$. Given that we have a *stock* pollutant, the $T1$ policy effect of the damage during $T1$ itself becomes vanishingly small, and it is easy to see that the effect on the future periods $T2$ is readily accounted for by the consideration of the value of the shift of the future (utility) time-path, ultimately implying that (14) still describes the optimal tax path, where the evaluation of the path of the U_ts and their time derivations with account for the pollution readily account for the pollution damage. Since the accumulating pollution implies an additional element with an increasing disutility into the utility flow function (16), $\dot{U}_t$ generally becomes more steeply downward-sloping (necessarily, e.g., in the no-policy benchmark) and thus implies generally higher equilibrium taxes in (14).

This extends as well to multiple buyers similarly to above, wherewith Prop. 1 extends:

Proposition B. The time-consistent unit tax of a buyer of a non-renewable resource that emits a pure-stock pollutant, extracted with increasing extraction cost and competitively supplied by a Stackelberg follower, potentially facing competition by a finite number $n \in [0, \infty)$ of Nash consistently competing and/or passive[12] buyers, follows

$$\tau_t^* = -\frac{\xi_{i,t}}{x_t} \int_t^\infty e^{-\rho(v-t)} \dot{U}_{i,v} \mathrm{d}v, \tag{17}$$

with $\xi_{i,t}$ the time t consumption share of competitor i, $\xi_{i,t} = x_{i,t}/x_t$.

3 Numerical illustration

We use a simple model to illustrate and confirm results from above, in a model for the global oil market. The model considers monopsonistic worldwide demand for oil, with a demand elasticity $\varepsilon = -0.9$, calibrated to today's price and quantity pair of 30.7 Gbbl/yr

[11] (per se here a function of the stock of emissions, this is, however, assumed to map directly to the resource stock)

[12] Governed by undistorted decentralized consumption.

and 76 \$/bbl (IEA, 2010; World Bank, 2011). The competitive resource owners can extract resources at a cost of 50 \$/bbl initially, rising linearly with cumulative extraction, by 50 \$/1000 Gbbl extracted (thought of as a reasonable rate of increase given that IEA, 2008, predicts a total of around 5500 Gbbl of oil left underground today). Subject to this cost curve, the suppliers price their fuel according to the FOCs in (4) and (5). The discount rate is $\rho = 6\,\%$, and the consumer utility is according to section 2.1. When we consider the total social welfare, this is defined as the sum of the consumer's welfare plus the (NPV of the) fuel supplier resource rent. The convergence to zero of the commitment duration is essential to Karp's assertion of the zero taxes. To make it as clear as possible that resulting positive taxes are not only artifacts of a coarse numerical approximation, we thus used time-step durations of only 0.3 years, using 500 time steps, i.e., a simulation-horizon of 150 years.[13]

Fig. 2 in Annex A.2 illustrates the fully competitive model outcome (no taxes), showing the time path of the oil consumption rate and consumer utility flow, market price and extraction costs, as well as the resource rent. Naturally, oil consumption and utility flow decrease over time, as explained in the previous chapter. The current value resource rent λ decreases as extraction costs rise over time,[14] and it drops to zero towards the end of the model horizon of 150 years. Fig. 3, comparing the path of the present value of the multiplier, Λ, for the 150-year horizon to that of a longer horizon, confirms that 150 years is by far long enough to have the relevant final present value shadow multiplier mentioned relevant in Karp's claim validly approximated as zero.[15]

Fig. 1 plots various calculated tax paths (or values) relevant for our case, zoomed in for the first 50 years. The left plot gives various estimates of different estimated tax paths. Dotted blue, $\tau_{consist.|0}$, gives the estimate of the consistent tax calculated with 14, using the utility values U_t calculated for the model outcome for $\tau_t = 0$. Solid red, in turn, gives the same tax, but for the case where we have iteratively calculated the tax according to 14 and used this tax to recalculate the model outcome with the corresponding U_t path that provides a (re-)estimate of the optimal tax path, until convergence (which was fast), giving the optimal consistent tax, $\tau^*_{consist.}$. Finally, the other two graphs in the left plot

[13]Simulations with varying the simulation duration have shown that this ensures that the ending of the simulation horizon has negligible effects on the results up to at least the first 100 years.

[14]Theoretically, while the present value rent Λ necessarily decreases over time, the current value (that is, not present value discounted) rent, λ, can also rise over parts of the considered time horizon, e.g. in models with different forms of the extraction cost curve.

[15]Fig. 3 shows that for longer time horizons, the present value multiplier Λ at the end of the here considered 150-year model horizon would in reality take on a value in the order of 0.001 \$/bbl instead of 0 \$/bbl as in our model. According to Karp, in presence of a non-zero final multiplier, the buyer could only set a tax that reduces the multiplier to just zero but leaves the extraction path to zero. It is easy to see that this would imply a tax that equals, in present value, the value of the final multiplier (cf. Eq. (6) and (4), and note that, for $\Lambda = \lambda \exp(-\rho t)$, Eq. (5) implies $\dot{\Lambda} = -e^{-\rho t}\dot{c}$). From this point of view, one could potentially claim a tax of in the order of 0.001 \$/bbl as explainable by the finite time horizon considered here. The consistent (initial) tax values found here are, however many orders of magnitude larger, exceeding 10 \$/bbl.

give the taxes that were numerically calculated directly as those which maximize the total welfare of the buyer (giving the tax path $\tau^*_{commit.,max}$) or the total social welfare (that is, buyer plus seller welfares; path $\tau^*_{social,max}$, solid gray line). The path of $\tau^*_{social,max}$ conforms to what we know since Hotelling's (1931) seminal contribution: the tax that maximizes total social welfare is zero. As expected further, there exists a substantial optimal import tax on the fuel, of around 20 \$/bbl and rising over time ($\tau^*_{commit.,max}$); by consuming less, the buyer improves his terms-of-trade by reducing the fuel import price. For our purpose, the most informative paths are, however, those for the optimal uncommitted tax, $\tau^*_{consist.}$, and for the theoretical optimal uncommitted tax calculated based on the equilibrium outcome without taxes, $\tau_{consist.|0}$. $\tau^*_{consist.}$ starts at a value slightly below the optimal committed rate,[16] $\tau^*_{commit.,max}$, but the two diverge substantially from each other over time, as was expected: the committed tax at $t > 0$ improves terms-of-trade also for previous periods, increasing the benefits from the tax and thus increasing the optimal rate, but it is exactly the peculiarity of the consistent tax that it 'cannot' account for influences on the past. Nevertheless, $\tau^*_{consist.}$ remains substantial and clearly differs from zero. Finally, even $\tau_{consist.|0}$, which would be nil according to Karp, as his claim for the optimal, consistent tax of the monopsonist facing competitive supply would be zero, and must thus be so independent of potential past taxes, are found to be substantial (only slightly lower than the optimal consistent tax, $\tau^*_{consist.}$. To be even more sure on that point, one may wish to consider the convergence of the welfare maximizing initial tax rate as the commitment period approaches zero from an initially longer time period. Whilst the here used model readily allows to directly find, numerically, the tax rate which, when sustained for a certain duration, leads to the largest buyer welfare, these durations can in the model used not be chosen arbitrarily small, as we are constrained to positive multiples of the time-step length, here set to 0.3 years, which thus gives the lowest directly inspectable commitment period. The curve in the right plot of Fig. 1 indicates the utility maximizing initial tax, as a function of for how long it is sustained (x-axis value). It becomes obvious that, even if this smooth curve is extrapolated linearly or nonlinearly to a commitment period of 0, from the lowest calculated value of 16.7 \$/bbl for a period of 0.3 years (black dot in the plot),[17] the value becomes not zero. Instead, it converges very closely to the initial value calculated analytically for the same scenario with zero future taxes (blue dotted in left plot and blue dot in right plot, 16.3 \$/bbl), confirming that Eq. 10 yields the welfare maximizing instantaneous tax under the assumption of future taxes being zero.

[16] Whilst in the graph it may look almost as if they converged for $t \to 0$; this is not really the case.

[17] Extrapolation to a zero-commitment period yields 16.6 \$/bbl.

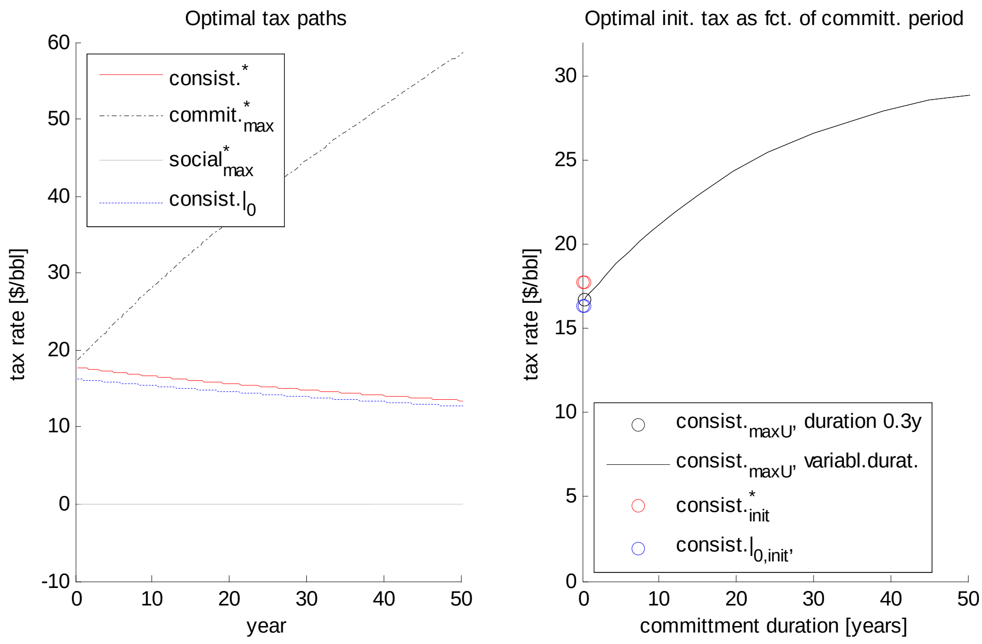

Figure 1: Tax rates

References

H. Hotelling (1931), The Economics of Exhaustible Resources, *Journal of Political Economy*.

IEA (International Energy Agency) (2008), World Energy Outlook 2008, OECD/IEA, Paris.

IEA (International Energy Agency) (2010), World Energy Outlook 2010, OECD/IEA, Paris.

L. Karp (1984), Optimality and Consistency in a Differential Game with Non-Renewable Resources, *Journal of Economic Dynamics and Control* 8, pp. 73 – 97.

World Bank (2011), World Bank Commodity Price Data (Pink Sheet).

Annex

A.1 Relation resource stock and buyer utility

The following shows that an increase by Δ in the stock S_t of the competitively supplied resource increases the NPV of future buyer's utility U_t by $d \cdot \Delta$ with a finite, non-marginal positive d.

Note: we use λ_t as the *current* value shadow value, contrary to Karp who denotes it λ^* and denotes the *present* value counterpart as λ_t.

Consider an initial situation, where at a normalized time $t = i$, an initial stock S_i is given. As we have no taxes from time i on, the outcome is given by the demand FOC, $u'(x_t) = p_t$, and the pricing equation, $p_t = c_t + \int_t^\infty e^{-\rho(s-t)} \dot{c}_s \mathrm{d}s$, where we directly accounted for $p_t = c_t + \lambda_t$ and, as we have a final resource unit value of zero, $\lambda_t = \int_t^\infty e^{-\rho(s-t)} \dot{c}_s \mathrm{d}s$. As is well known from the exhaustible resource literature, in this case, the price path exhibits a nonmarginal increase over time at least as long as a nonmarginal amount of (economically relevant) stock is still extractable[18], that is, for a nonmarginal duration d_1 from now on, we have a positive, nonmarginal time derivative of p_t: $\exists e > 0 \, s.t. \, \dot{p}_t > e \, \forall \, t \in [i, i + d_1]$, with nonmarginal e and d_1 (note, e and d_1 are not proportional to ΔS; instead they remain independent and non-marginal even for infinitely small ΔS). Standard exhaustible resource literature also teaches us that in standard cases we will have positive, non-marginal extraction rates throughout from time i through $i + d_1$, i.e., extraction is always larger than a specifiable, nonmarginal $x_{min} > 0$ during this phase.

Now, consider that the resource stock is increased by a marginal amount ΔS to $S_t^* = S_t + \Delta S$. In this case, since the extraction rate is finite, it will take a positive, nonmarginal multiple of ΔS of time, $d_2 = k\Delta S$ with a nonmarginal $k > 0$, until the ΔS additional units are consumed. From time $i + d_2$ on, nothing has changed compared to the initial situation at time t, besides the time shift, that is, from time $t + d_2$ on, the original price path results, shifted by d_2 into the future: we have $p^*_{t+d_2} = p_t$. In addition, for the period up to $t + d_2$, standard Hotelling price-monotonicity implies $p^*_{t \in [i, i+d_2]} < p_i$, as the additional reserves reduce the sales price. This implies that from i through d_2 we have a reduced price, and, moreover, during the nonmarginally long period $i + d_2$ through $i + d_2 + d_1$ we have a price that is nonmarginally reduced by at least the nonmarginal amount $e \cdot d_2$. Finally, also for times beyond $i + d_2 + d_1$ we at least have the weak inequality, $p_t^* \leq p_t$. The buyer utility flow is thus never decreased, but the time from $i + d_2$ through $i + d_2 + d_1$ implies an increase of the NPV value by at least $\int_{i+d_2}^{i+d_2+d_1} e^{-\rho t} x_{min} e \cdot d_2 \mathrm{d}t = x_{min} \cdot e \cdot k \cdot \Delta S \int_{i+d_2}^{i+d_2+d_1} e^{-\rho t} \mathrm{d}t$. For finite times i, all terms (besides the ΔS) in the last expression are non-marginal and

[18] This is the case at least if we restrict our attention to the case of a finite resource stock and a nonmarginal rate of increase of the marginal extraction cost as resource Stock depletes, $c'(S) < -z$, with a nonmarginal z.

positive factors. In conclusion, when the resource stock is increased by a marginal unit ΔS at a finite time i, this increases the utility of the (purely competitively acting) buyer by a an amount that is a nonmarginal multiple of ΔS.■

A.2 Details numerical model

Fig. 2 shows the general model results. Fig. 3 illustrates the evolution of the present value resource rents, Λ_t, for different model horizons. All simulations are based on a 0.3 year time-steps.

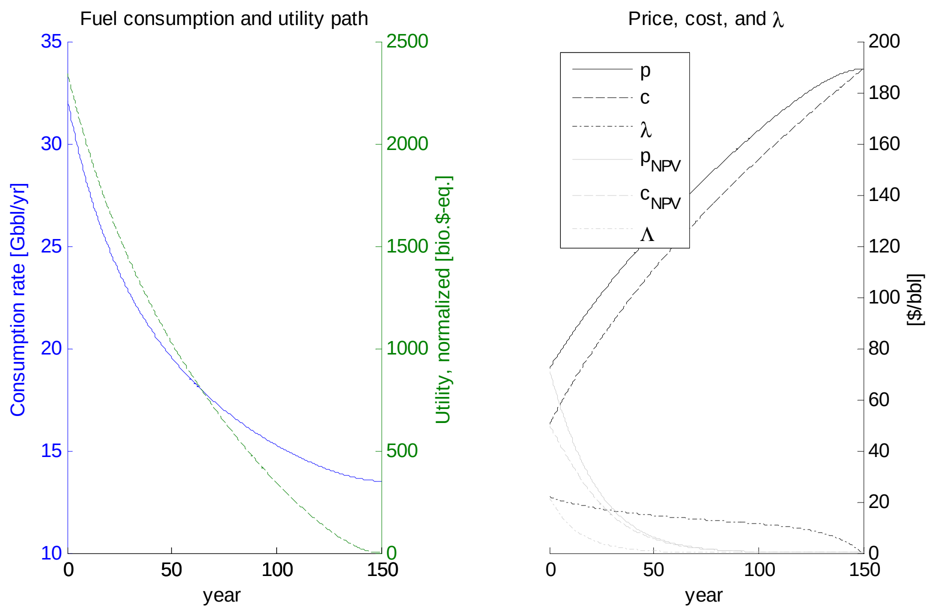

Figure 2: Main model results

Simulations in absence of taxes. In the right plot, Λ is the present value of the resource rent λ. The consumer utility (right axis in left plot) is shifted such as to become zero in the last period.

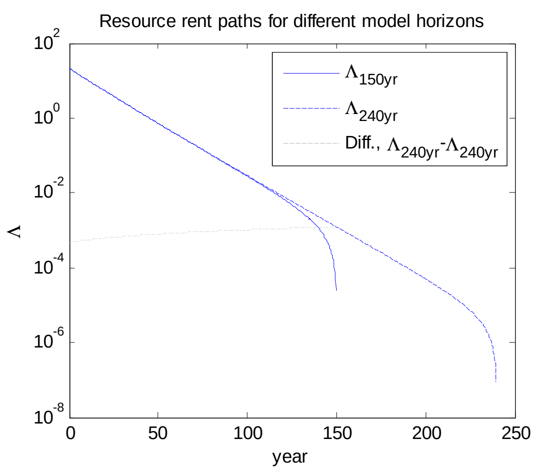

Figure 3: Shadow multipliers Λ_t

Simulations in absence of taxes.