Asymmetric Tax Competition:
Theoretical and Empirical Results

by

FLORIAN HABERMACHER

University of St. Gallen, Swiss Institute for International Economics
and Applied Economic Research (SIAW-HSG),

and

GEBHARD KIRCHGÄSSNER

University of St. Gallen, Swiss Institute for International Economics
and Applied Economic Research (SIAW-HSG), Leopoldina and *CES*ifo

Abstract

After presenting some preliminary casual evidence for Swiss cantons and member countries of the European Union, a small theoretical model of asymmetric tax competition is developed. It considers Nash and Stackelberg equilibria and extends previous work by allowing for a Leviathan government and considering the effects of a fiscal equalisation system. Finally, relations between pairs of adjoining cantons in Switzerland with strong competitive relations are considered. There are Granger causal relations from Zug to Zürich, from Appenzell Outer Rhodes to St. Gallen and, but only a very weak one, from Basel-County to Basel-Town. Thus, while in the first two examples the smaller canton behaves as a Stackelberg leader, between the two Basel the agglomeration canton is the Stackelberg follower.

Keywords: Tax Competition, Size, Agglomeration, Asymmetry, Stackelberg Solution, Swiss Cantons.

JEL Classification: H20, H87

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Mailing Address: Prof. Dr. GEBHARD KIRCHGÄSSNER
University of St. Gallen
SIAW-HSG, Bodanstrasse 8
CH-9000 St. Gallen
Switzerland
Gebhard.Kirchgaessner@unisg.ch

1 Introduction

[1] The core of really vivid federal systems, as, for example Switzerland, Canada, or the United States, is the right of sub-federal political units to decide not only about their expenditure but also about their own tax revenue, which necessarily results in tax competition. Moreover, tax competition is not only an issue with federal countries, but also in supra-national organisations as the European Union and also at the international level. The question for conditions of fair tax competition, as raised, for example, by OECD activities in the last decade, is, besides the problem of tax evasion, not only since the outbreak of the financial and economic crisis one of the most important ones if not the main issue in international negotiations about tax policy.[1)]

[2] If we consider the international situation, most problems arise when small countries have considerably lower taxes than larger ones. Most tax havens are typically (very) small countries, like Liechtenstein or Singapore, or small parts of larger countries that are, however, allowed to have their own tax regime, like the Channel Island belonging to the United Kingdom. Consequently, conflicts, if they arise, arise between large and small countries and rarely between large ones.

[3] Problems of size are, however, not the only asymmetries that play a role in federal systems. The main problem of Canada is, in this respect, that one province, Alberta, has additional revenue from oil which the other provinces do not have, i.e. it is financially in a totally different situation and can, therefore, offer considerably lower taxes than the other provinces. Moreover, the new economic geography starting with P. KRUGMAN (1991, 1991a) as well as the recent discussion about institutions versus geography as drivers of economic growth point to additional asymmetries which can, in federal countries, have an effect on their tax policy.

[4] These problems are, however, hardly taken into account in the economic literature about fiscal federalism. More than 50 years ago, CH. TIEBOUT (1956) started the economic theory of federalism with his seminal paper. Since then, many papers have been produced about tax competition, theoretical as well as empirical ones.[2)] However, contrary to the political experience, the vast majotity of the theoretical papers employ models of symmetric tax competition; two (and sometimes more than two) identical countries are considered and effects of variations of, for example, different tax instruments are discussed. There are only few papers that explicitly consider asymmetric situations, in particular the conflict between a large and a small country and it is shown that in such a situation the smaller one will have lower taxes but more tax revenue per capita than the larger one.[3)] This result is in strong contrast to the idea of CH. TIEBOUT (1956) that tax payers choose those locations with those combinations of public

1. See for this, for example, OECD (1998, 2006) as well as TH. RIXEN (2005).
2. See, for example, the surveys of W.E. OATES (1999, 2001, 2005), but also L.P. FELD (2000).
3. See, for example, R. KANBUR and M. KEEN (1993), V. ARNOLD (2001), CARDARELLI, E. TAUGOURDEAU and J.-P. VIDAL (2002), S. PERALTA and T. VAN YPSERLE (2005, 2006), or J. HINDRIKS, S. PERALTA and S. WEBER (2008), but in particular S. BUCOVETSKY (1991), J.D. WILSON (1991), A. HAUFLER (2001, pp. 74ff.), R.E. BALDWIN and P. KRUGMAN (2004) as well as N. CHATELAIS and M. PEYRAT (2008).

services and taxes which mostly correspond to their preferences and which would imply that political communities with lower tax rates also have lower tax revenue as well as public expenditure per capita.

[5] The empirical papers also hardly ever explicitly consider asymmetric situations. Because real world situations are never symmetric, sometimes asymmetry indicators like the size of a local community or the degree of urbanisation are included as explanatory variables. However, the strategic options for tax policy resulting from such asymmetries are hardly ever discussed when the results are presented.[4)]

[6] That problems connected with asymmetric tax competition are largely neglected in the economic literature has also implications for the discussion of fiscal equalisation systems. As is well known, such systems provide disincentives to take care of their tax base for both, those who receive money from other communities as well as those who have to give away some of their own tax revenue to others. The literature concentrates on these negative effects and hardly discusses the positive effects of such systems. Of course, at least in some countries like, for example, Germany, the negative effects dominate, but this does not contradict the fact that such a system might be necessary in many asymmetric situations.[5)]

[7] This paper attempts to have a closer look at the consequences of asymmetric tax competition under different circumstances. We mainly consider those asymmetries caused by differences of size. The paper is organised as follows: We first show some empirical facts for Switzerland and the European Union which provide casual evidence for the impact of asymmetric tax competition which is, at least partly, against conventional wisdom (*Section 2*). In *Section 3*, we develop a simple theoretical model. There, we not only consider Nash, but also Stackelberg equilibria. Moreover, we do not only cover situations of benevolent, but, following the ideas of G. BRENNAN and J.M. BUCHANAN (1977, 1980), also of Leviathan governments. In addition, we also investigate the effects of fiscal equalisation systems.[6)] *In Section 4*, we look at pairs of adjoining Swiss cantons in asymmetric situations with strong interactions: Zürich and Zug, St. Gallen and Appenzell Outer Rhodes, as well as the two Basel. The empirical evidence suggests that in these cases the canton with the lower tax rates is the Stackelberg leader, which is not necessarily the smaller one. We conclude with some remarks on the implications of our results for the Swiss fiscal federal system. (*Section 5*)

[8] When discussing problems of asymmetric tax competition, one should, however, distinguish between tax havens, as discussed, for example, in J. SLEMROD and J.D. WILSON (2009) or J. BECKER and C. FUEST (2010),[7)] on the one side and small countries on the other side.

4. There are, however, several papers estimating the effects of agglomerations. See, for example, R. BORCK and M. PFLÜGER (2006), S. COULIBALY (2008), H.-J. KOH and N. RIEDEL (2010) or J. JOFRE-MONSENY (2011).

5. See for this, for example, M. KÖTHENBÜRGER (2002) or C. GAIGNÉ and S. RIOU (2007).

6. Throughout the main part of the paper we focus on an implementation of the model with simple functional forms. A more general version, based on the approach of J.D. WILSON (1991) and R.E. BALDWIN and P. KRUGMAN (2004) is examined in the *Appendix*.

7. See also M.A. DESAI, C.F. FOLEY and J.R. HINES Jr. (2006, 2006a).

While tax havens (only) provide possibilities for profit shifting and hardly attract production factors, small countries with low tax rates, besides also providing possibilities for profit shifting, also attract production factors.[8)] This holds for the smaller member countries of the European Union and also for Switzerland, whereas Liechtenstein with its 64'996 firms and only 36'150 inhabitants and an area of 160 km^2 might be assessed as being a tax haven.[9)] In this paper, we do not consider tax havens but small countries or federal states (cantons) which try to attract production factors by offering favourable tax conditions.

2 Some Preliminary Empirics

[9] Following CH. TIEBOUT (1956), the traditional argument is that citizens, following their preferences, choose between communities with high taxes and high public expenditure, and communities with low taxes and low public expenditure. Among the well known assumptions of this model is not only that people have identical incomes but that those two (or more) communities have identical characteristics. The Swiss reality shows, however, quite another picture. Take the two adjoining cantons, Zürich and Zug. Today, Zürich has about 1.3 million inhabitants, Zug about 110'000. The two capitals of these cantons are only 36 km away from each other; during the day there are eight hourly trains between Zürich and Zug, the fastest taking only 21 minutes. Thus, there is no problem to work in Zürich and to live in Zug.

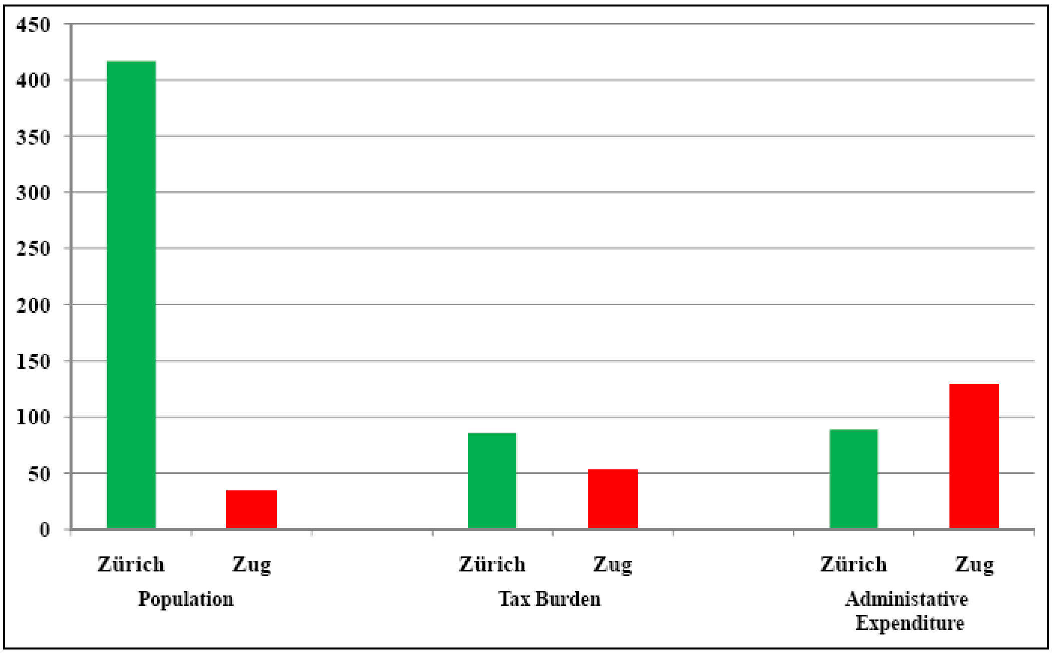

Figure 1: Population, Tax Burden and Administrative Expenditure per capita in the cantons Zürich and Zug, 2006. (Swiss Average = 100)

8. L.P. FELD and G. KIRCHGÄSSNER (2003) show, for example, that tax competition between the Swiss cantons does not only lead to profit shifting but also to a re-allocation of jobs.

9. This was the situation as of end of 2010. See: http://www.llv.li/llv-as-bevoelkerung; http://www.llv.li/amtsstellen/llv-gboera-oera/llv-gboera-oera-amtsgeschaefte-statistik.htm (30/01/12).

[10] About 40 years ago, Zug started a low tax policy. Today it has very low personal income taxes for high income earners and also very low profit taxes. As *Figure 1* shows, the tax burden in Zug is much lower than in Zürich. Public Expenditure per capita are, however, rather high in Zug, and, in particular, administrative expenditure, are among the highest in Switzerland.[10] The low taxes attracted so many rich people and (letterbox) companies that public expenditure can be rather high. Moreover, as the high administrative expenditure show, it is definitely not the parsimony of the public administrations which allows the low tax burden.[11]

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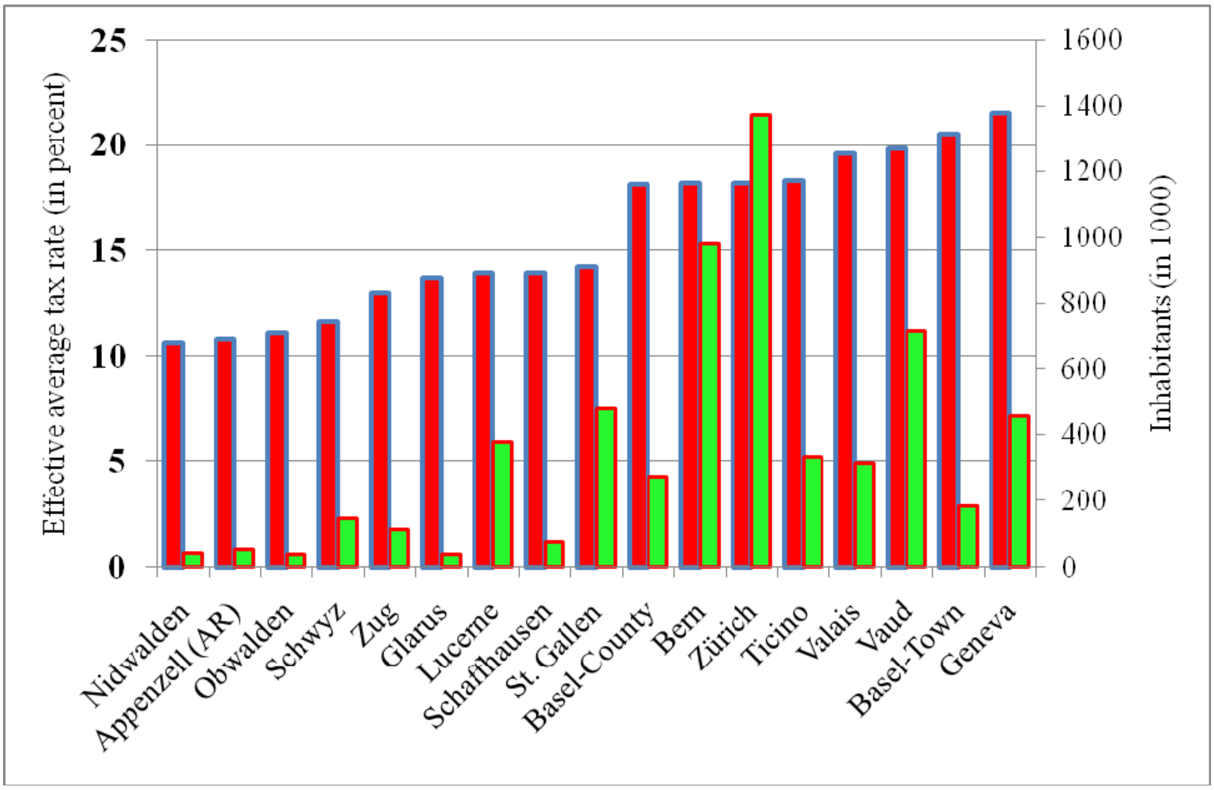


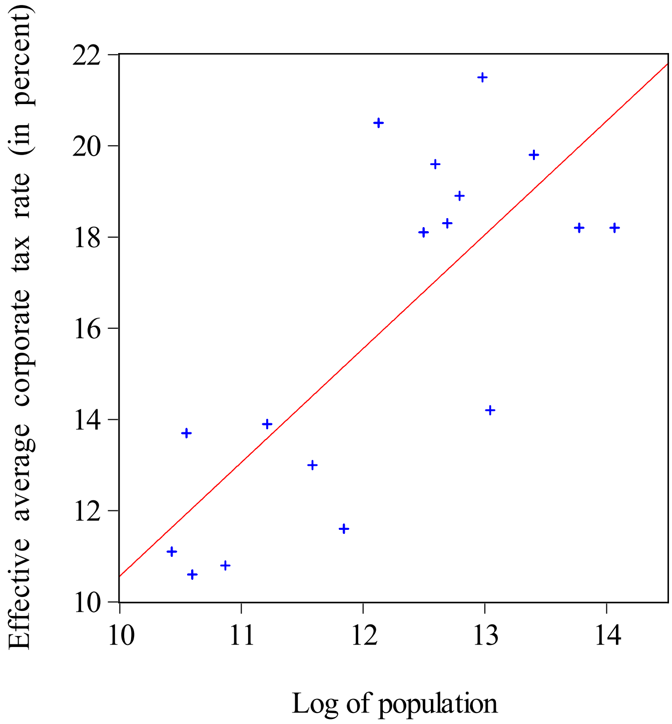
Figure 2: Effecting Average Tax Rates (in percent) 2011 for Companies and Population of Selected Swiss Cantons

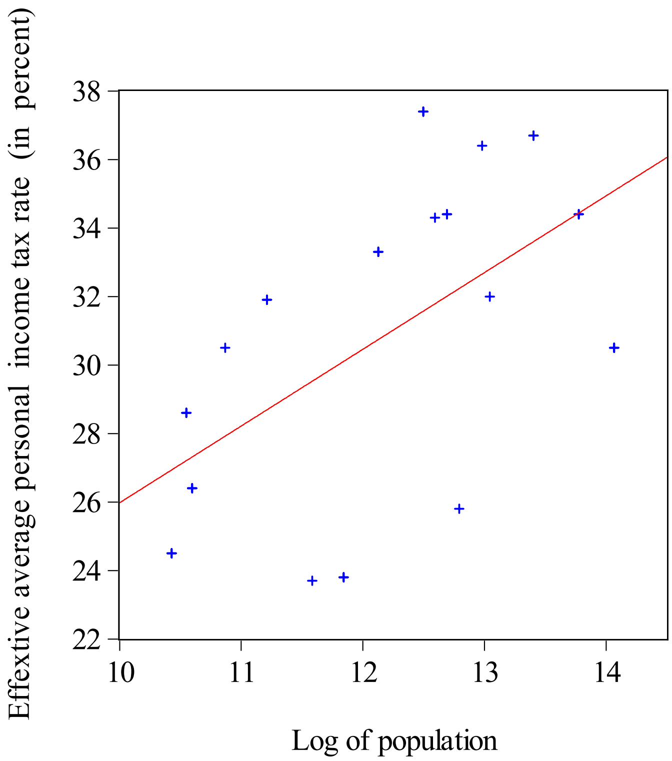
[11] To see whether there is a more general relation between the size of a canton and its tax burden we use data from the BAK Taxation Index 2011.[12] The results do not only indicate

10. The data are for 2006, because this is the latest date for which the Federal Financial Administration in Bern published data for the cantonal tax burdens. The date are normalised for the Swiss average = 100. – Source of the data: *Statistisches Jahrbuch der Schweiz 2011*, Electronic Version, Table j-d-01.02.01.0101, j-d-18.02.02.03.11; EIDGENÖSSISCHES FINANZDEPARTEMENT, *Öffentliche Finanzen der Schweiz 2006*, Bundesamt für Statistik, Neichâtel 2008, p. 126.

11. It is more meaningful to compare public administration expenditure instead of total expenditure per capita because the tasks of the public administration are more or less the same in all cantons whereas total expenditure depend very much on the topographical and socio-economic characteristics of the cantons. In addition, we take cantonal and local expenditure together because the task distribution between these two levels differs very much between the cantons.

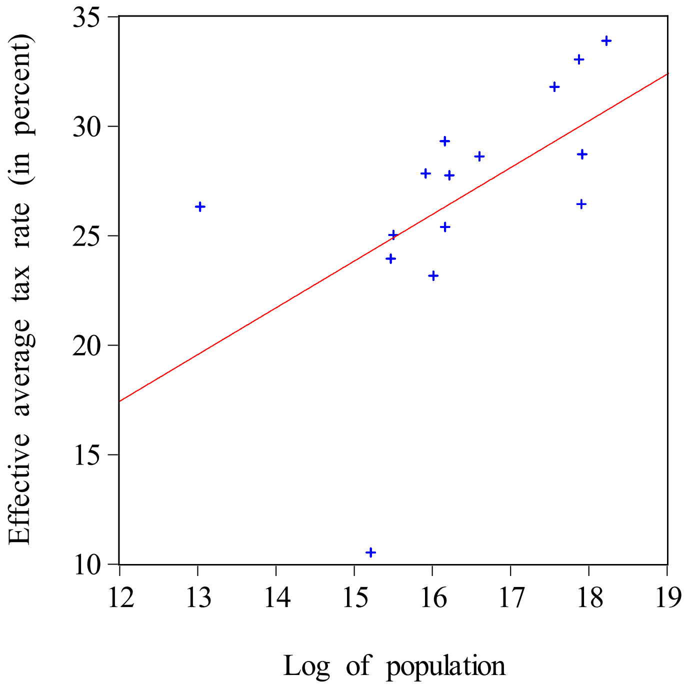
12. The BAK Taxation Index is produced by BAK Basel and the Centre of European Economic Research in Mannheim (ZEW). They calculate effective average and marginal tax rates for companies as well as effec-





tive tax burdens on highly qualified manpower for a sample of industrial countries and 17 Swiss cantons. The data we use are from the Executive Summary of the BAK Taxation Index 2011, http://www.bakbasel.ch/downloads/competences/location_factors/taxation/BAK_Taxation_Index_2011_Executive_Summary_E.pdf (30/01/12).

Figure 3: Population and Effective Tax Burden in 17 Swiss Cantons, 2011



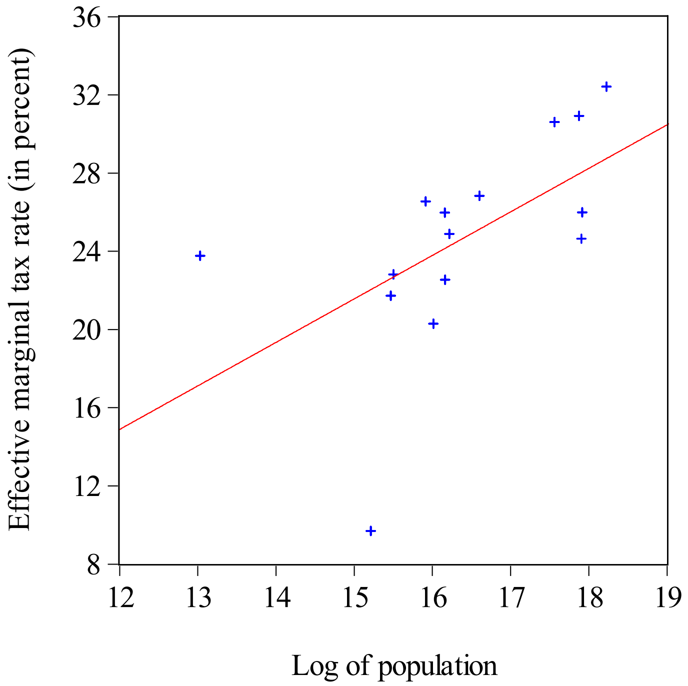


Figure 4: Population and effective tax rates in the 15 old member countries of the European Union, 2000 – 2007

hat business taxation in Switzerland is low in comparison to other countries: of those locations considered only Hong Kong has a lower effective average tax rate than Nidwalden, and besides Hong Kong only Dublin has a lower effective average tax rate than St. Gallen (but higher than Schaffhausen). *Figure 2* shows the effective average tax rates in 2011 for companies of the 17 Swiss cantons that are included in the sample of this index. It indicates that the smaller cantons have lower tax rates. *Figure 3* shows the relation between the logarithm of the populations of these 17 cantons and their tax burden. In both cases we find a clear positive relation: The larger the population, the higher the tax burden. For the effective average tax rate of companies (in percent), the correlation is 0.763 and statistically significant at the 0.01 percent level; for the tax burden on manpower the correlation is 0.556 and statistically significant at the 5 percent level.

[12] We find similar results for the member countries of the European Union. *Figure 4* shows the relation between the populations of the 15 old member countries and the effective average and marginal tax rates, calculated with the method proposed by M.P. DEVEREUX and R. GRIFFITH (1999). The data are averages over the period from 2000 to 2007. The picture is clear: there is a strong positive relation between the size of the country and the average tax rates, with correlations of 0.53 for the average and 0.57 for the marginal tax rates. We get similar pictures with only slightly reduced correlations if we include those four new member countries for which data are available, and with somewhat more reduced correlations if we include the other OECD countries; in the latter case we get a correlation of 0.48 for the average and of 0.41 for the marginal tax rates. Nevertheless, these (statistically significant) results are very clear, for the national as well as the international level: compared to large units, small units have generally lower tax rates in systems with strong tax competition. Moreover, these correlations increased dramatically over time, for the 15 old EU members from 0.28 and 0.30 in 1990 via 0.44 and 0.47 in 2000 to 0.58 and 0.59 in 2007 for the effective and marginal tax rates, respectively. At the same time, the (unweighted) average of the tax rates of these 15 countries decreased from 0.326 to 0.245 for the effective average and from 0.328 to 0.224 for the effective marginal tax rates, respectively. Thus, the Swiss as well as the European situation suggests that the relation between the size of a governmental unit and its tax burden deserves a deeper investigation.[13)]

3 A Simple Model of Asymmetric Tax Competition

[13] As mentioned above, there are only very few papers taking into account different country sizes. The model used here follows N. CHATELAIS and M. PEYRAT (2008), which is a variant of the basic model presented by S. BUCHOVETSKY (1991) and J.D. WILSON (1991) and further elaborated by A. HAUFLER (2001, Chapter 5).[14)] The progress of N. CHATELAIS and M. PEYRAT (2008) is that, by investigating the situation among the member countries of the European Union, they do not only consider the Nash equilibrium but also the situation of

13. See also the results of RR. HERNÁNDEZ-MURILLO (2003) who shows that, in the United States, not only the tax rates of neighbouring states but also the size of the population of a state has a highly significant positive impact on its capital tax rate.

14. See also R.E. BALDWIN and P. KRUGMAN (2004). The more general theoretical model presented in the *Appendix* is mainly based on J.D. WILSON (1991) and on this paper.

Stackelberg competition with the small countries as leaders and the large countries as followers.

[14] We have two countries, i = 1, 2, with representative firms with production functions

(1) $$X_i = F(L_i, K_i),$$

with constant returns to scale and L_i being the national labour force which is immobile and K_i being the internationally mobile capital endowment. The firms maximise the profit function

(2) $$\Pi_i = f(k_i) - r\,k_i - t_i\,k_i,$$

with r being the common interest rate, k_i being the capital intensity and t_i the (proportional capital tax rate in country i. Profit maximisation requires

(3) $$f'(k_i) = r + t_i,$$

and the arbitrage condition requires

(4) $$f'(k_1) - t_1 = f'(k_2) - t_2 = r.$$

The capital intensities are given by

(5) $$\bar{K} = K_1 + K_2 = k_1 L_1 + k_2 L_2,$$

with the total amount of capital, $\bar{K}$, as well as the national labour supplies being fixed. The average capital intensity as well as the individual endowments are given by

(6) $$k^* = \bar{K}/(L_1 + L_2).$$

To make the analysis simple, we use identical quadratic production functions in both countries,

(7) $$f(k_i) = \beta\,k_i - \alpha\frac{k_i^2}{2}, \quad \text{with } \alpha, \beta > 0.$$

Thus, from (4) and (5) it follows that the capital intensities in the two countries are

(8a) $$k_1 = \frac{t_2 - t_1}{\alpha}s_2 + k^*,$$

(8b) $$k_2 = \frac{t_1 - t_2}{\alpha}s_1 + k^*,$$

where s_1 and s_2 are the (fixed) shares of the labour force in the two countries $s_i = L_i/(L_1 + L_2)$, and the interest rate is

(9) $$r = \beta - t_1 s_1 - t_2 s_2 - \alpha\,k^*.$$

The income of the citizens, composed of capital and labour income, is

(10) $$x_i = w_i + r\,k^* = f(k_i) - k_i\,f'(k_i) + r\,k^*.$$

Taxes are used to finance the public good

(11) $$g_i = t_i\,k_i\,.$$

The general formulation of the utility function of the citizens is given by

(12) $$U_i(f(k_i) - k_i f'(k_i) + r k^*, t_i k_i).$$

In order to further facilitate the analysis we assume additive linear preferences,

(12') $$U_i(x_i, g_i) = x_i + \gamma g_i,$$

with $\gamma > 1$.[15)]

3.1 The Nash Solution

[15] Under these assumptions, the problem of a benevolent government is to maximise the utility function

(12') $$U_i(f(k_i) - k_i f'(k_i) + r k^* + \gamma g_i) = x_i + \gamma g_i$$

subject to the constraints (8). From this we get the two reaction functions

(13a) $$t_1 = \frac{\gamma - s_2}{2\gamma - s_2} t_2 + \alpha \frac{\gamma - 1}{s_2(2\gamma - s_2)} k^*,$$

(13b) $$t_2 = \frac{\gamma - s_1}{2\gamma - s_1} t_1 + \alpha \frac{\gamma - 1}{s_1(2\gamma - s_1)} k^*.$$

If the two countries are of equal size, i.e. if $s_1 = s_2 = 0.5$, this reduces to

(13') $$t_1 = \frac{\gamma - 0.5}{2\gamma - 0.5} t_2 + \alpha \frac{\gamma - 1}{\gamma - 0.25} k^*,$$

and we get the corresponding equation for the tax rate of the second country. This leads to the Nash equilibrium solution

(13'') $$t_1 = t_2 = \frac{\gamma - 1}{\gamma} 2 \alpha k^*.$$

Equations (13) show that any increase in the tax rate of one country will allow the other country to increase its taxes as well. If, on the other hand, the sizes are different, we get the following solution for the tax rates:

(14a) $$t_1 = -\frac{(\gamma - s_2)s_2 + 2\gamma s_1 - s_1^2}{s_1 s_2 (\gamma(s_2 - 3\gamma) + \gamma s_1)} (\gamma - 1) \alpha k^*,$$

(14b) $$t_2 = -\frac{(\gamma - s_1)s_1 + 2\gamma s_2 - s_2^2}{s_1 s_2 (\gamma(s_1 - 3\gamma) + \gamma s_2)} (\gamma - 1) \alpha k^*.$$

or, if we substitute $s_2 = 1 - s_1$,

15. This condition is necessary in order to get an interior solution. Otherwise, the optimal tax rates would be zero.

(14'a) $$t_1 = \frac{(\gamma-1+s_1)(1-s_1)+2\gamma s_1 - s_1^2}{s_1\gamma(1-s_1)(3\gamma-1)}(\gamma-1)\alpha k^*,$$

(14'b) $$t_2 = \frac{(\gamma-s_1)s_1+2\gamma(1-s_1)-(1-s_1)^2}{s_1\gamma(1-s_1)(3\gamma-1)}(\gamma-1)\alpha k^*.$$

For $s_1 = s_2 = 0.5$ it holds that

(15a) $$\frac{\partial t_1}{\partial s_1} > 0, \quad \frac{\partial t_2}{\partial s_1} < 0.$$

Moreover, it generally holds that

(15b) $$\frac{\partial t_1}{\partial s_1} > 0 \quad \text{and} \quad \frac{\partial [t_1 - t_2]}{\partial s_1} > 0,$$

i.e. the larger the large in relation to the small country, the larger the spread will be between the two tax rates. If the size of the small country goes to zero, the tax rate as well as the welfare of the large country will approach the cooperative solution. Propositions I.1 through I.3, in the *Appendix* show that these results extend to more general conditions than those imposed here, i.e. without the specific functional form assumption (7).

[16] This is just a version of the famous theorem of S. BUCHOVETSKY (1991) and J.D. WILSON (1991):

If the two countries differ only in size, then the smaller country levies the lower capital tax rate and has the higher per capita utility level in the asymmetric Nash equilibrium.[16)]

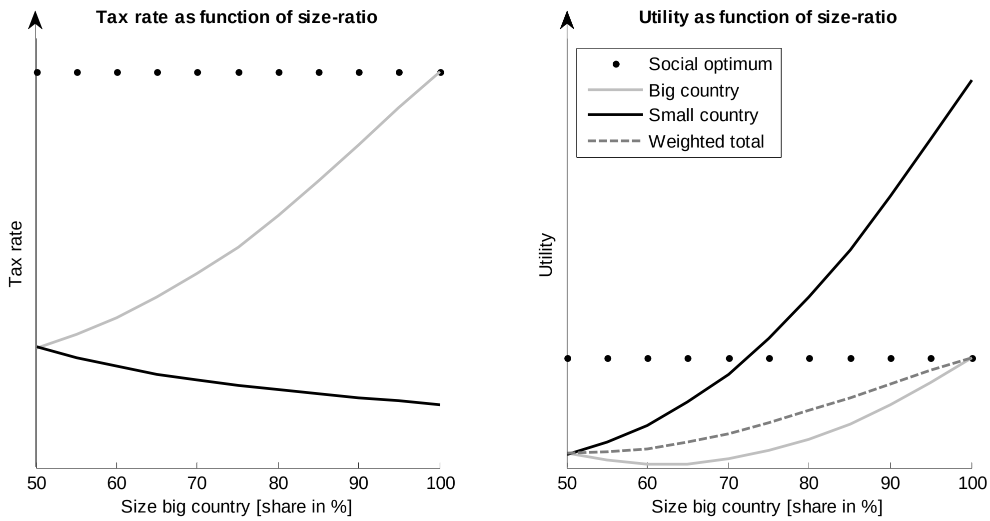


Figure 5: Equilibrium tax rates and corresponding utilities for asymmetric country sizes, simulations for separable logarithmic utility

16. This is the formulation given in A. HAUFLER (2001, p. 78). – M. WREDE (2009) shows that this also holds under formula apportionment.

Figure 5 shows this with a numerical example. Deviating from the theoretical model we have, however, assumed a logarithmic utility function for the simulation in order to limit the optimal tax rate.

[17] At a first glance, this result might be interpreted as a hint that tax competition is the more intense the more equal in size the different competitors are. According to this, the competition between France and Germany should be more intense than the one between these two countries and Switzerland. We observe, however, just the opposite. An explanation for this puzzle is given if we recognise that the difference in size between the two counties has to be large enough so that the small country is benefitting from lowering its taxes; it has to be relatively small so that the revenue losses from lowering the tax rate are more than offset by the additional revenue due to the enlargement of the tax base. Otherwise, none of the countries has an incentive to deviate from the cooperative equilibrium. Thus, even if Switzerland is competing with these two countries, they both have an incentive to build a coalition, and the three-party game reduces to the two-party game analysed above.

[18] This leads us to the second famous theorem of S. BUCHOVETSKY (1991) and J.D. WILSON (1991):

> *If differences in country size between two otherwise identical jurisdictions are continuously increased, there must be a critical distribution of the world population when the smaller country has a higher per capita utility level in the non-cooperative Nash equilibrium, as compared to the equilibrium with coordinated tax rates.*[17]

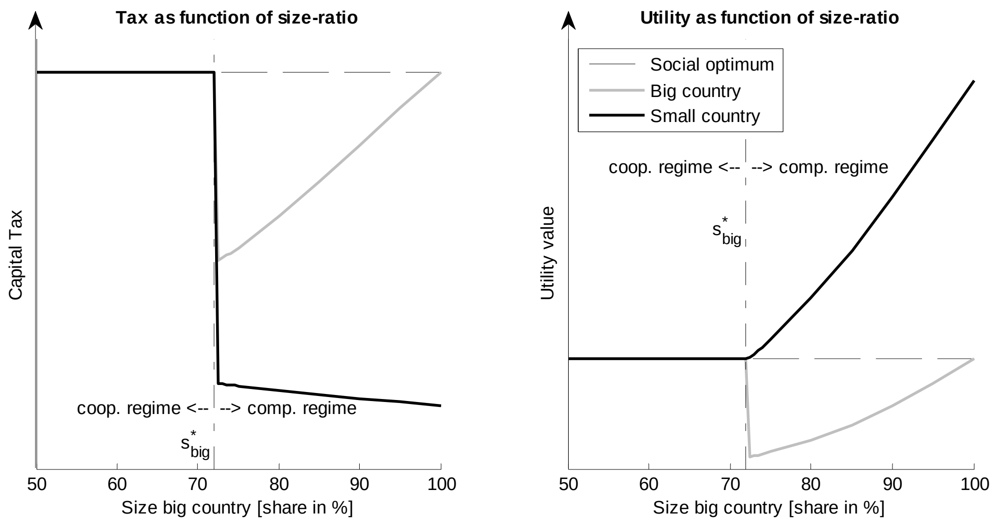


Figure 6: Equilibrium tax rates and corresponding utilities for asymmetric country sizes when the countries cooperate as long as this benefits even the small country

Figure 6 shows the corresponding regions again with a numerical example. If the country-size difference is small, both countries get a higher utility if they cooperatively set their tax rates to the socially optimal values (cooperative regime, left parts of the plots). If the small country

17. This is again the formulation given in A. HAUFLER (2001, p. 78).

is small enough, it has a larger utility when deviating, leading to the competitive outcome (right parts of the plots). Proposition I.3 in the *Appendix* establishes that this holds also in a generalised framework without the specific functional forms assumed above.

[19] The question is, of course, how important these effects are. One obvious limitation of the model is that only one tax instrument is considered. Thus, A. HAUFLER (2001, pp. 80ff.) develops a model of asymmetric tax competition where the two countries have two tax instruments: the capital tax and a wage tax. Simulations show that in such a situation only very small countries would benefit from tax competition; in his example this would only be the case if the small country had less than eight percent of the world's population. Thus, negative effects of tax competition should not be a very important problem, and the incentive to cooperate should be rather strong for the larger countries, even if one has to take into account that there are at least some tax havens which hardly ever would be willing to join such a cooperation.

[20] The problem with this model is, however, that it assumes labour to be immobile. As soon as we assume that labour is also mobile, i.e. that two mobile factors are taxed, the old problem arises. To show this, the model presented above can be used with only some minor modifications. Assume that land is the fixed factor, that capital is internationally fully mobile and can be used with a constant interest rate, and that labour is the production factor which is mobile inside a certain region, as, for example, a federal country or the European Union. Then, the same model applies as above, but now wage taxes will be lower in the smaller and higher in the larger country.

[21] We get similar results if we deviate from the assumption of a benevolent government and presuppose a Leviathan government. The reaction functions are

(16a) $$t_1 = \frac{1}{2}t_2 + \alpha \frac{1}{2(1-s_1)}k^*,$$

(16b) $$t_2 = \frac{1}{2}t_1 + \alpha \frac{1}{2s_1}k^*.$$

If the two countries are of equal size, i.e. if $s_1 = s_2 = 0.5$, this reduces to

(16') $$t_1 = \frac{1}{2}t_2 + \alpha k^*,$$

and we get the corresponding equation for the tax rate of the second country. This leads to the Nash equilibrium solution for the tax rates

(17') $$t_1 = t_2 = 2\,\alpha\,k^*,$$

which are, of course, higher than the tax rates in the Nash equilibrium with benevolent governments. Again, any increase in the tax rate of one country will allow the other country to increase its taxes as well. If, on the other hand, the sizes are different, we get the following solution for the tax rates:

(18a) $$t_1 = \frac{1+s_1}{3s_1(1-s_1)}\alpha k^*,$$

(18b) $$t_2 = \frac{2-s_1}{3s_1(1-s_1)}\alpha k^*.$$

Relation (15a) and (15b) hold again, i.e. the larger the large country in relation to the small country, the larger the spread will be between the two tax rates. The welfare consequences are, however, now ambiguous. Because the Leviathan government overtaxes the citizens, some reduction of the tax burden is beneficial, and it is open whether the reduction due to the tax competition will be so large that their welfare will be smaller compared to the exploitation equilibrium realised by cooperative Leviathan governments. Proposition II in the Appendix shows this ambiguity again for the case of more general functions.

3.2 The Stackelberg Solution

[22] So far, only Nash equilibrium constellations have been considered. The situation in Switzerland but also in the European Union is, however, not an equilibrium one. Over the last decades, single countries and/or cantons started new rounds in the game and it were typically small countries which did this. This is understandable, given the results derived above that small communities will be the potential gainers in this battle while the large communities will necessarily lose. N. CHATELAIS and M. PEYRAT (2008) were the first ones to model this as a Stackelberg game, where the small country is the leader. If we adopt this idea for our model, under the assumption of a benevolent government, we get for the (small) leader canton the following solution

(19a) $$t_2^{leader} = -\frac{2\gamma^2(2-s_1) - \gamma(4-7s_1+4s_2^2)+s_2^3}{s_1(1-s_1)\gamma^2(4\gamma+s_1-2)}(\gamma-1)\alpha k^*,$$

which results in a higher tax rate compared to the Nash equilibrium. Thus, the (large) follower canton, having still (13a) as its reaction function, with

(19b) $$t_1^{follower} = \frac{\gamma((2-s_1)-1+s_1)(\gamma-1+s_1)^2}{s_1(1-s_1)\gamma^2(4\gamma+s_1-2)(3\gamma-1)}(\gamma-1)\alpha k^*,$$

as its equilibrium solution, will also have a higher tax rate. If, on the other hand, the two cantons play sequentially Nash, they will finally end up in the Nash equilibrium and both will be worse off than in the Stackelberg solution. Proposition III in the Appendix confirms that this holds also for the more general model formulation.

[23] *Figure 7* demonstrates this situation. We have the two reaction functions, and the leader picks the point on the follower's reaction function which provides him the highest utility. Qualitatively, the situation looks similar in the case of a Leviathan government, and as long

as the small county is the leader, both countries again have larger tax rates in the Stackelberg equilibrium than in the Nash equilibrium.

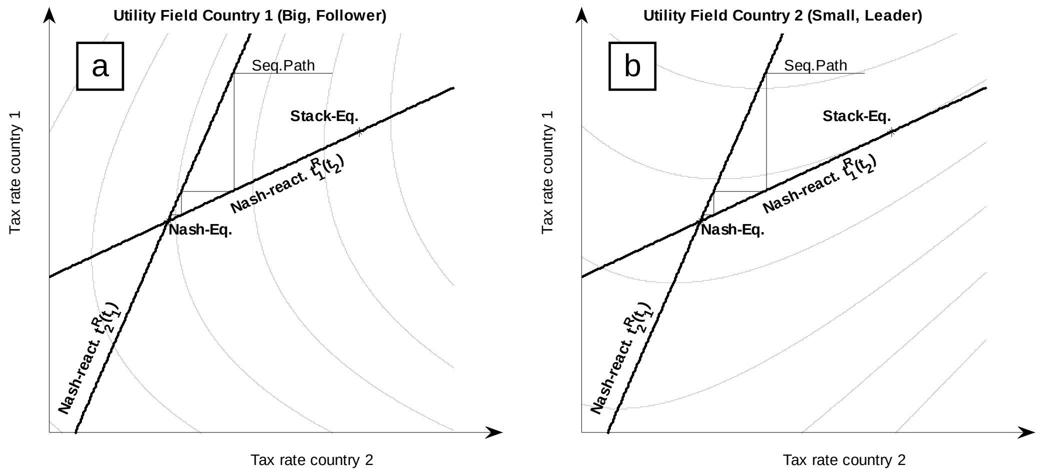


Figure 7: Concave utility fields (grey) for benevolent (plots a and b) and Leviathan (plots c and d) governments, and corresponding Nash reaction curves and Nash and Stackelberg equilibria. $t_i^R(t_j)$ ***is the Nash reaction of country i on the other country j's tax choice.***

3.3 The Fiscal Equalisation System

[24] A fiscal equalisation system redistributing globally some of the benefits from domestic capital attraction can be readily modelled as a tax on domestic capital to be paid by both governments and the equalisation tax receipts redistributed to the two countries by a simple weighting according to the regions' population shares. Writing t_f for the corresponding equalisation tax rate, the public revenue available net of the equalisation system writes

(20) $$G_i = t_i k_i + t_f [k^* - k_i],$$

and this is now also the amount of public good which enters the regional utility, that is, we redefine the utility as

(21) $$U_i(x_i, G_i) = x_i + \gamma G_i.$$

[25] In this case, the Nash reaction functions are

(22) $$t_i = \frac{\alpha k^*(\gamma-1) + \gamma s_j(t_j + t_f) - s_j^2 t_j}{s_j(2\gamma - s_j)},$$

where j indexes the competitor of country i. The equilibrium taxes become

(23) $$t_i = t_f + \alpha k^*(\gamma-1)\frac{(\gamma - 1 + s_i(2s_j + \gamma))}{\gamma(3\gamma-1)s_1 s_2},$$

where the second term on the RHS is nothing else than the Nash equilibrium tax rate in the standard case without equalisation system (equations (14)). Thus, we get the following theorem:

If under a fiscal equalisation system governments are charged an amount t_f per unit of capital employed in their countries and the corresponding receipts are distributed equally among the countries according to their population shares, the Nash equilibrium tax rates in the two countries increase in the equalisation tax rate t_f.

In the *Appendix* it is confirmed that the validity of this result extends also to a more general model formulation. With Propositions IV.1 and IV.2 it is established that for more general preferences with decreasing marginal utilities there exists an equalisation parameter t_f that leads to the socially optimal taxes and consumption.

Figure 8 illustrates this result in an example based on a logarithmic utility function.[18)] The example is based on a size ratio $s_1 : s_2 = 8$. Because of the strong asymmetry, only the large country gains from the equalisation scheme while the small country loses. The overall welfare defined by the sum of the countries' utilities weighted by their population shares, is, however, steadily increasing in the equalisation tax.

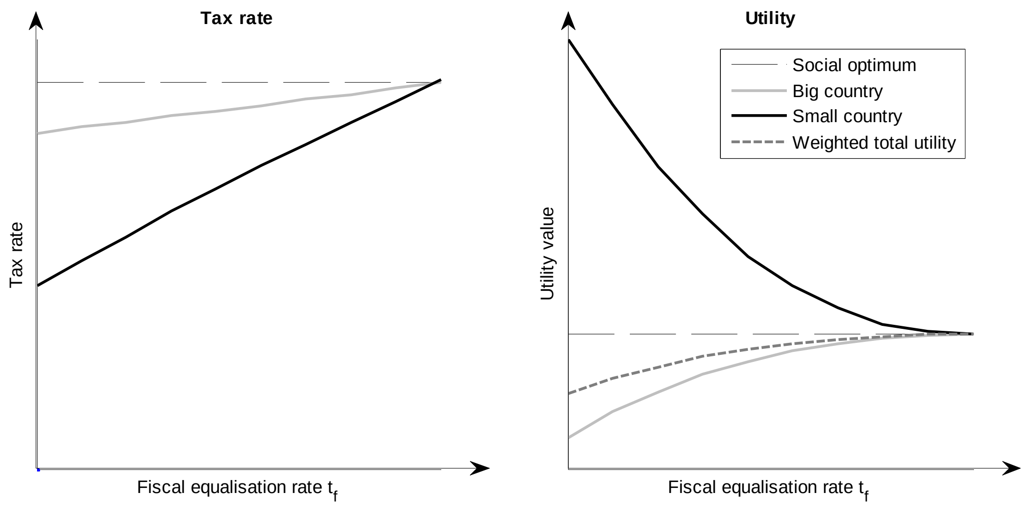


Figure 8: Tax rates and utilities with a fiscal equalisation system

4 Empirical Results for Some Swiss Cantons

[26] It is well known that there is intense tax competition between the Swiss cantons, and also between local communities within the same canton,[19)] but questions of asymmetry in size

18. While in the formula we derived based on the linear preferences, the equilibrium tax rates in both countries increase one-per-one with the equalisation tax t_f, *Figure 8* illustrates that the increase may differ between the two countries if non-linear utilities are assumed. However, the taxes are always increasing in the equalisation rate.

19. See, for example, G. KIRCHGÄSSNER and W.W. POMMEREHNE (1996), L.P. FELD and G. KIRCHGÄSSNER (2001, 2003), K. SCHMIDHEINY (2006) and L.P. FELD and E. REULIER (2008).

or agglomeration have not yet been investigated. To investigate this we select three pairs of cantons which are adjoining and among which intense competition exists. These are first of all Zürich and Zug, the two ones we already used as example. As mentioned above, in the seventies, the Canton of Zug was the first one which started a low-tax policy. It was mainly directed against its neighbour Zürich, the largest Swiss canton. The difference in the population of these two cantons is 12.5 to 1. Zürich reacted by reducing its tax rates as well, but, as predicted by our theoretical model, the tax rates of Zürich always remained higher than those of Zug. We find a similar situation between St. Gallen and Appenzell Outer Rhodes. There, the difference in size is 'only' 8.5 to 1. At a lower level than to Zürich, the city of St. Gallen can also be considered being an agglomeration, at least compared to Appenzell Outer Rhodes (and, of course, also to Appenzell Inner Rhodes). Again, St. Gallen has consistently higher tax rates than Appenzell Outer Rhodes. The third pair we look at are the two Basel, Basel-Town and Basel-County. There, the situation is somewhat different, because the smaller canton, Basel-Town has the higher tax rate. The difference in the population size between the two cantons is, however, very small, only 1.4 to 1. Thus, the smaller canton is hardly small enough to benefit from the tax competition game. On the other hand, Basel-Town might profit from its being an agglomeration which, according to the considerations of the 'New Economic Geography' allows to tax away, at least partially, agglomeration rents,[20)] while the (perhaps) only possibility of Basel-County to counteract its disadvantage from being a more peripheral canton is to attract people and/or firms by lower tax rates.[21)]

[27] *Figure 3* shows the average income tax rate for taxable incomes of 200'000 CHF for the period from 1983 to 2003 in the capitals of these three pairs of cantons.[22)] In all three cases tax rates were going down. This is, however, only due to effects of inflation and real income increases. Despite the fact that these taxes are indexed against inflation, between 1990 und 2007 the share of cantonal and local income and property taxes increased from 10.5 to 11.4 percent of GDP. Remarkable is, however, that we observe in all three cases clear differences between the two cantons, where the smaller canton (Zug and Appenzell Outer Rhodes) or the more rural canton (Basel-County) has consistently lower taxes rates over the whole observation period, the largest difference being between Zürich and Zug.

[28] To study more closely the dynamics between these cantons we first performed Granger causality tests for the tax rates. As already mentioned, all tax rates have a downward trend,

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20. For empirical evidence with respect to business taxation in Germany and Spain see H.-J. KOH and N. RIEDEL (2010) and J. JOFRE-MONSENY (2011), respectively.

21. There is an important difference between Zürich (and St. Gallen) on the one and Basel-Town on the other side. The canton of Basel-Town is nearly identical with the city of Basel, whereas in Zürich and St. Gallen there are large rural areas besides the urban centre. In the canton Basel-Town, 88.5 percent of the population live in communities with more than 30'000 inhabitants, compared to 27.3 percent and 15.3 percent in the cantons Zürich and St. Gallen, respectively. Thus, the agglomeration plays a much larger role in Basel compared with Zürich or St. Gallen. Source of the Data: *Statistisches Jahrbuch der Schweiz* 2011, Electronic version, Tables je-d-01.02.01.01.01 and je-d-01.02.01.01.02.

22. Source of the Data: EIDGENÖSSISCHE STEUERVERWALTUNG, *Steuerbelastung in der Kantonshauptorten*, *http://www.estv.admin.ch/dokumentation/00075/00076/00720/index.html?lang=de* (31/01/12).

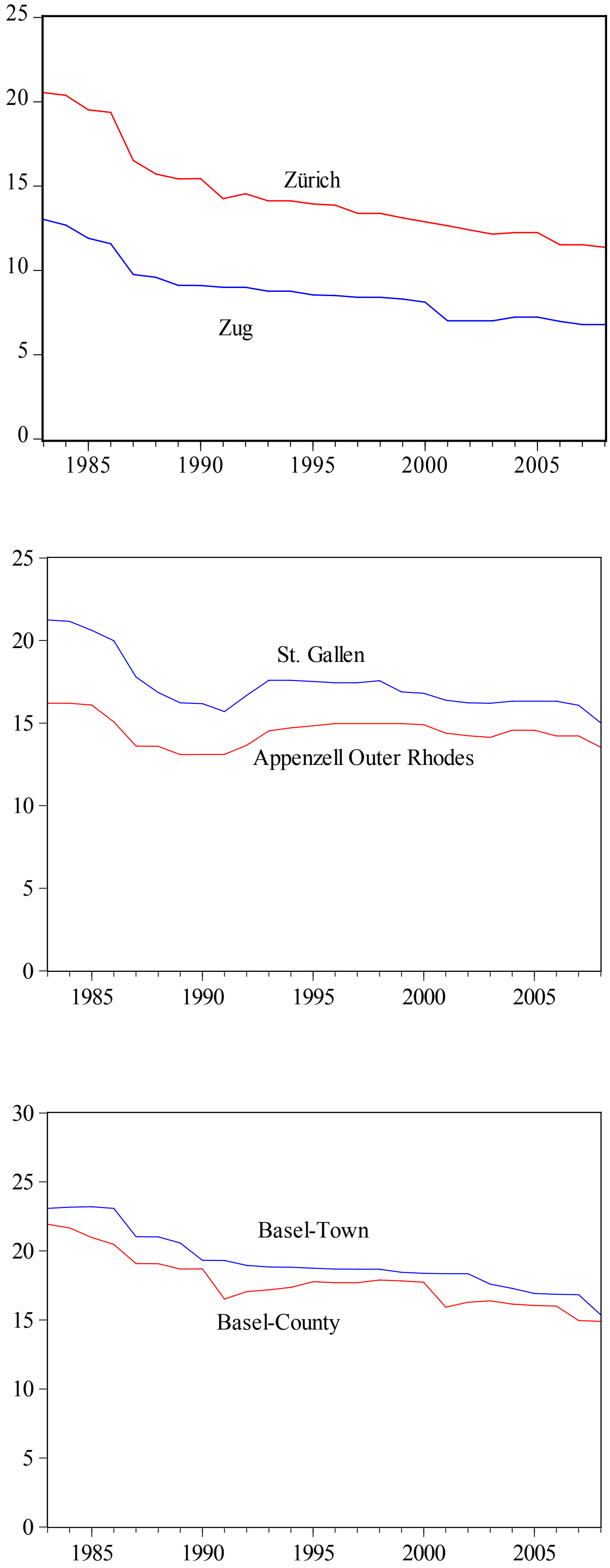

Figure 5:   Tax Rates (in percent) for High Income Earners for Selected Swiss Cantons, 1983 – 2008

even if this does not imply that the tax burden has been reduced over time. However, due to the small number of observations it is difficult to decide whether this is a deterministic or a stochastic trend; the power of the corresponding tests is too low. Thus, we performed the Granger causality tests for the levels as well as the first differences of the tax rates.

Table 1: Pairwise Granger Causality Tests
1984 – 2008

	Lags	$\widehat{F}$	p-values
ZG → ZH	1	4.400*	0.048
ZH → ZG		0.130	0.722
ZG – ZH		29.004***	0.000
ΔZG → ΔZH	1	1.652	0.213
ΔZH → ΔZG		0.473	0.499
ΔZG – ΔZH		27.580***	0.000
AR → SG	2	1.980	0.166
SG → AR		0.152	0.860
AR – SG		25.585***	0.000
ΔAR → ΔSG	1	4.224(*)	0.053
ΔSG → ΔAR		0.034	0.855
ΔAR – ΔSG		38.838***	0.000
BL → BS	2	2.697(*)	0.093
BS → BL		1.776	0.196
BL – BS		1.852(*)	0.081
ΔBL → ΔBS	2	1.198	0.325
ΔBS → ΔBL		0.650	0.534
ΔBL – ΔBS		0.994	0.333

[29] If the two cantons play a Stackelberg game, it is natural to assume that the leader is also a leader in time. Starting from the situation where both have (approximately) the same tax rates, only the smaller player has an incentive to deviate because the larger cantons can only lose by deviating from this equilibrium, while the smaller one, as shown above, can win. Thus, we expect to find a simple Granger causal relation from the tax rate of the smaller canton to the one of the larger canton, but no reverse relation.[23] There might, however, also be an

23. Granger causality tests to check who the Stackelberg leader in a tax competition game is are also employed by R. ALTSHULER and T.J. GOODSPEED (2006). Moreover, the assumption that the Stackelberg leader is moving first (in time) has also be employed by N. CHATELAIS and M. PEYRAT (2008) who show that – subject to this assumption – the small countries in the European Union are Stackelberg leaders.

instantaneous relation. The reason for this is that in the budgetary process, which takes place yearly, the cantons (can) observe each other and, therefore, already react in the same tax period.

[30] The results are given in *Table 1*.[24] For the two pairs Zürich (ZH) − Zug (ZG) and St. Gallen (SG) − Appenzell Outer Rhodes (AR) we have a highly significant instantaneous relation between the two cantons, independent of whether we use the levels or the first differences of the variables. In addition, using levels, we have a simple causal relation from Zug to Zürich at the 5 percent significance level, and, using first differences, the same relation from Appenzell Outer Rhodes to St. Gallen, significant at the 10 percent level, but no indication al all of a simple Granger causal relation in the opposite direction. Thus, if there is a Stackelberg game, the smaller canton should be the leader.

[31] The results for the two Basel are somewhat different. Using first differences, we find no relation at all. Using levels, we find an instantaneous relation and a simple causal relation from Basel-County (BL) to Basel-Town (BS), both being, however, only significant at the 10 percent level. Thus, if there is a Stackelberg leader at all, it is the more peripheral canton, Basel-County, and not the agglomeration, Basel-Town.

[32] As a final step, we estimated error correction models for the Stackelberg followers. We started with AR(1)-representation and included a linear trend to allow for a deterministic trend. It was, however, only significant in the St. Gallen equation. Thus, we dropped it from the other equations. We also dropped the not significant lagged values. In all cases we assumed that the instantaneous relation goes from the Stackelberg leader to the follower.

[33] The results including the long-run relations are given in *Table 2*. Given the high values of the adjusted multiple correlation coefficients, it is obvious that the tax rates of St Gallen and Zürich are to a large degree influenced by the tax policy of Appenzell Outer Rhodes or Zug, respectively. What is, however, more important are the significance of the lagged levels in these equations. It shows that the tax rate in the large cantons react to tax changes in the Stackelberg leaders in the expected direction. What might be astonishing is the insignificance of the constant terms in the error-correction as well as in the long-run relations. The reason for this might be that, given the short sample, the null hypothesis of a random walk (without thrift) cannot be rejected for the differences between the tax rates: the p-values of the Dickey-Fuller test statistic are 0.177 for the difference between Zürich and Zug and 0.395 for the difference between St. Gallen and Appenzell Outer Rhodes.[25] On the other hand, the estimated coefficients for the tax rates of the leader cantons reflect the different tax levels between the leader and the follower cantons. The results for the two Basel are somewhat different: the estimated coefficients of the leader canton as well as the adjusted multiple correlation coefficient are much lower. Again, the constant terms in the two equations are not significantly dif-

24. All calculations have been performed with EViews, Version 7.

25. We get, on the other hand, highly significant constants in estimated ARMA(1,1) models for these differences. In both cases we have, however, very high positive AR- and negative MA-coefficients, and even using conventional t-values the MA-coefficients are never significantly different from -1.0.

ferent from zero, despite the fact that the difference seems to be stationary: the corresponding p-value of the Dickey-Fuller statistic is 0.018.

Table 2: Error Correction Equations
1984 – 2008

Zürich and Zug

$$\Delta AITR(ZH)_t = \underset{(1.37)}{0.609} + \underset{(5.39)}{1.036}\,\Delta AITR(ZG)_t - \underset{(-3.36)}{0.538}\,AITR(ZH)_{t-1}$$

$$+ \underset{(3.28)}{0.804}\,AITR(ZG)_{t-1} + \hat{u}_t$$

$\bar{R}^2 = 0.726$, SER = 2.481, D.-W. = 2.102, J.-B. = 0.026, DF = 21.

Long-run relation:

$$AITR(ZH)_t = \underset{(1.37)}{1.132} + \underset{(17.323)}{1.496}\,AITR(ZG)_t$$

St. Gallen and Appenzell Outer Rhodes

$$\Delta AITR(SG)_t = \underset{(-0.40)}{-0.446} + \underset{(6.81)}{0.977}\,\Delta AITR(AR)_t - \underset{(-3.22)}{0.572}\,AITR(SG)_{t-1}$$

$$+ \underset{(3.31)}{0.767}\,AITR(AR)_{t-1} - \underset{(-2.65)}{0.062}\,TR_t + \hat{u}_t$$

$\bar{R}^2 = 0.780$, SER = 0.291, D.-W. = 1.822, J.-B. = 0.061, DF = 20.

Long-run relation:

$$AITR(SG)_t = \underset{(-0.39)}{-0.780} + \underset{(10.05)}{1.340}\,AITR(AR)_t - \underset{(-7.18)}{0.108}\,TR_t$$

Basel-Town and Basel-County

$$\Delta AITR(BS)_t = \underset{(-0.54)}{-0.515} + \underset{(1.86)}{0.296}\,\Delta AITR(BL)_t - \underset{(-3.32)}{0.513}\,AITR(BS)_{t-1}$$

$$+ \underset{(3.28)}{0.572}\,AITR(BL)_{t-1} + \hat{u}_t$$

$\bar{R}^2 = 0.262$, SER = 0.456, D.-W. = 2.171, J.-B. = 2.992, DF = 21.

Long-run relation:

$$AITR(BS)_t = \underset{(-0.53)}{-1.003} + \underset{(10.59)}{1.115}\,AITR(BL)_t$$

The numbers in parentheses are the t-statistics of the estimated parameters. SER denotes the standard error of regression, D.-W. the Durbin-Watson test-statistic for autocorrelation of the estimated residuals, J.-B. the Jarque-Bera test-statistic for normality of the estimated residuals, and DF the numbers of degrees of freedom of the t-statistics.

5 Summary and Conclusions

[34] If size is the only (or at least the most important) difference between two communities between which there is tax competition, and if this difference is large enough, the smaller one will have lower taxes but possibly higher tax revenue per capita. This prediction by a small theoretical model is fulfilled if we consider the two Swiss cantons Zürich and Zug. Moreover, in Switzerland, but also in the European Union and in the OECD, we find that the larger cantons or member countries, respectively, have higher tax rates. On the other hand, if we compare the two Basel which are not very different in size, Basel-Town with its agglomeration has higher taxes than the more rural canton of Basel-County. This is in line with the predictions of the New Economic Geography that agglomeration rents can, at least partially, be taxed away.[26)]

[35] Using Granger causality tests, we also checked whether these cantons play Nash or a Stackelberg game. Behind this procedure there is the (plausible) assumption that, if any one, the Stackelberg leader moves first in time. For the Swiss cantons we find a similar result as N. CHATELAIS and M. PEYRAT (2008) for the European Union: if there are differences in size, the smaller units are the Stackelberg leaders. If we investigate the two Basel, the agglomeration canton is the Stackelberg follower. Thus, in all three cases considered the canton with lower taxes is the Stackelberg leader.

[36] Size and agglomeration are, of course, not the only reasons for asymmetries which cause tax rate differentials. Differentials in resources are another candidate. The obvious example is the Canadian province Alberta, which, due to its oil revenue, is able to have lower taxes than the other Canadian provinces. Another example is Germany in the second half of the 19th century. As M. HALLERBERG (1996) shows, Prussia with its revenue from the nationalised railways was the Stackelberg leader. In this case, the additional revenue from the railways overcompensated the incentives due to size differences.

[37] As the theoretical model shows, the Stackelberg solution implies higher tax rates than the Nash equilibrium and is, therefore, compensating at least part of the welfare loss due to tax competition. Moreover, as the theoretical model also shows, the existence of a fiscal equalisation system leads to a convergence of the tax rates. In Switzerland, since 2008 there is a new fiscal equalisation system in force which is more effective that the previous one.[27)] Given the evidence presented above that in Switzerland Stackelberg games are played, this system should raise tax rates. On the other hand, the system of direct democracy in the Swiss cantons with its fiscal referenda and debt brakes should insure that governments cannot behave like Leviathans.[28)] Thus, the Swiss federal fiscal systems has checks and balances which should assure that tax levels are not too far away from those of the social optimum.

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26. See, for example, R.E. BALDWIN and P. KRUGMAN (2004). – Another indication for the possibility to tax away agglomeration rates is the fact that the city of Zürich as well as the city of St. Gallen have higher tax rates than their surrounding local communities which have the right to set their own tax rates.

27. For the new Swiss fiscal equalisation system see, for example, G. KIRCHGÄSSNER (2006).

28. See for this, for example, L.P. FELD and G. KIRCHGÄSSNER (2000).

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Analytical Appendix

A.1 Model Description

The model corresponds in general to the description in Section 3, but we abstain from assuming specific functional forms and instead rely on some standard properties.

We have two countries, indexed with $i \epsilon \{1, 2\}$. Labour supply L_i is assumed to be fix in each country, with s_i expressing the population (or labour) share of country i in the global labour supply,

$$s_i \equiv \frac{L_i}{\Sigma_j L_j},$$

which implies that we can write

$$s_2 = 1 - s_1. \tag{A.1}$$

Both countries have the same production function given in its intensive form, $f(k_i)$, expressing the output per worker, where k_i is the capital intensity of production, $k_i \equiv K_i/L_i$. As standard conditions we impose $\infty > f'(k_i) > 0 > f''(k_i) > -\infty$.

The capital *endowment* (not to be confounded with the capital 'intensity' employed in production in a country, k_i) of each worker in both countries is the same, denoted by k^*. The total amount of capital in the whole economy is fixed, so that we have

$$k^* = s_1 k_1 + s_2 k_2. \tag{A.2}$$

For given net capital rental rate r (interest rate) and capital tax rate t_i, the firm profit function in country i is defined as

$$\Pi_i = f(k_i) - rk_i - t_i k_i.$$

Profit maximization with respect to capital employed then implies

$$f'(k_i) = r + t_i. \tag{A.3}$$

As the capital owners choose freely in which country to invest their capital, the net-of-tax interest rate r is the same in both countries and the following arbitrage condition derives from (A.3):

$$f'(k_1) - t_1 = f'(k_2) - t_2 = r. \tag{A.4}$$

As the population in one region remains the owner of the capital even if it invests it abroad, private income x_i is $x_i = w_i + rk^*$, where rk^* is capital income and $w_i = f(k_i) - k_i f'(k_i)$ the labour income. We thus rewrite private income (or private consumption) as

$$x_i = f(k_i) - k_i f'(k_i) + rk^* \tag{A.5}$$
$$= f(k_i) + [k^* - k_i] f'(k_i) - t_i k^*. \tag{A.6}$$

The public good g_i financed with the tax on domestically *employed* capital is

$$g_i = t_i k_i. \tag{A.7}$$

Partial derivatives of (A.5) and (A.7) with respect to the domestic tax rate are

$$\frac{\partial x_i}{\partial t_i} = -k_i f''(k_i)\frac{\partial k_i}{\partial t_i} + \frac{\partial r}{\partial t_i}k^* \tag{A.8}$$

$$= [k^* - k_i]f''(k_i)\frac{\partial k_i}{\partial t_i} - k^* \tag{A.9}$$

$$\frac{\partial g_i}{\partial t_i} = k_i + t_i\frac{\partial k_i}{\partial t_i}. \tag{A.10}$$

Using (A.2) in (A.4) yields

$$\frac{\partial k_i}{\partial t_i} = \frac{1}{f''(k_i) + \frac{s_i}{1-s_1}f''(k_j)} = \frac{s_j}{s_j f''(k_i) + s_i f''(k_j)}. \tag{A.11}$$

The regional utility function (assumed to respect standard conditions and to be well-behaved) is the same in both countries,

$$U(x_i, g_i) = U(f(k_i) - k_i f'(k_i) + rk^*, t_i k_i), \tag{A.12}$$

for which we assume positive but decreasing returns with respect to both arguments:

$$U_{cc} < 0 < U_c \, \forall c \epsilon \{x, g\}.$$

In order to prevent longer examinations of the exact shape relations required for inner solutions to exist we require the derivatives of U to be finite, except when the consumption of one of the goods approaches zero, where we impose

$$\lim_{x\to 0} U_x = \infty \wedge \lim_{g\to 0} U_g = \infty. \tag{A.13}$$

To simplify the analysis we assume in general that the problem is well-behaved enough for there to exist only one Nash equilibrium (see also the notes in the proof for Lemma 3 for a rationale for this assumption). We further assume that the optimal tax rates leave positive net capital earnings (except in the case of the Leviathan government which, absent competition, necessarily sets the tax as high as external restrictions allow), that is, for optimal t_i and corresponding capital movements, we have $r = f'(k_i) - t_i > 0$. This seems a logical condition for at least two reasons: first, it is not conceivable that private actors employ additional capital if they have a net loss from it. Second, whilst in the here applied model capital is exogenous, it is in reality the result of private agents' production decisions. Those actors would not produce so much capital as to drive net rental prices to zero.

For the remainder, we define the marginal rate of substitution, MRS, as the marginal utility of public consumption relative to that of private consumption,

$$\text{MRS} \equiv \frac{U_g}{U_x},$$

and the marginal rate of transformation with respect to any variable v (this variable v will typically be a country i's tax, t_i), as

$$\text{MRT} \equiv \frac{-\partial x/\partial v}{\partial g/\partial v},$$

that is, the amount of private consumption that must be given up for a marginal increase of public consumption.

Note that the first-order optimality condition for a country's free choice of variable v, where we denote the optimal value as v^*, can then be expressed with the Samuelson rule

$$v^* = v \,\text{such that}\, \text{MRS}|_v = \text{MRT}|_v. \tag{A.14}$$

A.2 Model Analysis

For what follows, we write t^* for the socially optimal tax rates, which the countries may decide to implement if they cooperate. In the standard competitive setting, $t_{i,N}$ designates the Nash equilibrium tax rate of country i. Having in the symmetric situation with $s_1 = s_2$ that $t_{1,N} = t_{2,N}$, we write this symmetric equilibrium tax rate as t_N. In the competitive setting with a Leviathan government we index the competitive Nash equilibrium variables with an L, writing t_L for the symmetric case.

A.2.1 Standard Situation

A.2.1.1 Symmetric Countries

Consider two countries of the same size, i.e.,

$$s_1 = s_2 = 0.5. \tag{A.15}$$

Cooperation

In the cooperative equilibrium where both countries set the same tax levels, $t_1 = t_2$, they also find the same amount of capital employed, $k_1 = k_2 = k^*$, that is, the capital intensity in production does not change when the countries change their cooperatively set tax rate.

Using (A.4) (arbitrage) in (A.5) yields in this case

$$x_i = f(k^*) - tk^*.$$

Utility, (A.12) becomes

$$U(x, g) = U(f(k^*) - t \cdot k^*, t \cdot k^*). \tag{A.16}$$

In (A.16) we see that the marginal rate of transformation between public and private good, defined as MRT $\equiv -\frac{\partial x/\partial t}{\partial g/\partial t}$, equals 1. Having the substitution counterpart, MRS $\equiv U_g/U_x$, we therewith have the following simple optimality condition for the optimal cooperative tax rate t^*, derived from the Samuelson rule (A.14):

$$\frac{U_g(x(t^*), g(t^*))}{U_x(x(t^*), g(t^*))} = 1.$$

This result is intuitive: labour and capital employed are fixed. The tax thus transfers private income in a one-per-one ratio into public revenue, wherewith the optimum split between the goods is where the marginal consumption utility is the same for both goods.

Nash competition

Consider now instead the case where the two countries set their taxes competitively. In this case, the following Proposition holds:

Proposition I.1

The Nash equilibrium tax rate of the two competing countries of similar size is below the socially optimal level. Public good is underprovided and utility levels are below those in the cooperative setting.

Proof of Proposition I.1:

We consider the case of symmetric countries, that is, $s_i = 0.5$. Using (A.2) and (A.15) in (A.4) yields:

$$f'(k_1) - t_1 = f'(2k^* - k_1) - t_2 = r. \tag{A.17}$$

The optimal reaction of t_1 on a fixed t_2 is still characterized by the Samuelson rule (A.14), i.e.,

$$\frac{U_{g,1}}{U_{x,1}} = -\frac{\partial x_1/\partial t_1}{\partial g_1/\partial t_1}. \tag{A.18}$$

For a fixed t_2, the first equality in the arbitrage condition (A.17) implies

$$\mathrm{d}f'(k_1) - \mathrm{d}f'(2k^* - k_1) = \mathrm{d}t_1. \tag{A.19}$$

Note thus that, at the equilibrium, i.e., for $k_1 = k_2 = k^*$, we have $\mathrm{d}f'(k_i) = f''(k^*)\mathrm{d}k_i$, and because of $\mathrm{d}k_2 = -\mathrm{d}k_1$, expression (A.19) yields $2f''(k^*)\mathrm{d}k_1 = \mathrm{d}t_1$, i.e. we have

$$\mathrm{d}k_1 = \underbrace{\frac{1}{2f''(k^*)}}_{<0}\mathrm{d}t_1. \tag{A.20}$$

Inserting (A.20) directly in the original arbitrage condition, (A.4), yields

$$\mathrm{d}r = \frac{\mathrm{d}t_1}{2f''(k^*)}f''(k_1) - \mathrm{d}t_1 = -\frac{1}{2}\mathrm{d}t_1. \tag{A.21}$$

Using (A.20) and (A.21) in (A.8) yields, for an open economy in the symmetric competitive equilibrium,

$$\mathrm{d}x_1 = -k^*\mathrm{d}t_1. \tag{A.22}$$

As can directly be seen in (A.16), (A.22) holds as well in the autarkic (or, correspondingly, in the symmetric cooperative) situation where the capital employment is fixed at k^*. We thus conclude[1]

$$\left.\frac{\partial x_1}{\partial t_1}\right|_{t_2=t^*,\text{open}} = \left.\frac{\partial x_1}{\partial t_1}\right|_{t^*,\text{autarky}}. \tag{A.23}$$

Further, using relation (A.20), while recalling that we would have $k_1 = k^* \forall t_1$ under autarky, in (A.10) implies

$$\left.\frac{\partial g_1}{\partial t_1}\right|_{t_2=t^*,\text{open}} < \left.\frac{\partial g_1}{\partial t_1}\right|_{t^*,\text{autarky}}. \tag{A.24}$$

Note that in the *cooperative* equilibrium with tax rates $t_1 = t_2 = t^*$ we have

$$\underbrace{\frac{U_{g,i}}{U_{x,i}}}_{\text{MRS}_i} = \underbrace{-\frac{\partial x_i/\partial t_i}{\partial g_i/\partial t_i}}_{\text{MRT}_i} = \underbrace{-\partial x_i/\partial t_i}_{>0}\frac{1}{\partial g_i/\partial t_i} = 1,$$

[1] A possible intuition, why $\partial x_1/\partial t_1|_{t_2=t^*}$ does not differ between the autarkic and the competitive situation is that even though reducing the tax attracts – in the competitive situation – foreign capital and therewith increases labour productivity (compared to a tax reduction in the autarkic situation), the capital's marginal productivity decreases due to the inflow of foreign capital (again compared to a tax reduction the autarkic situation). Because, in addition, the capital rents of the additional capital attracted flow to the other country (the foreign country remains the *owner* of inflowing capital in the model), the net effect of the openness for a marginal tax change starting from an equilibrated tax $t_1 = t_2 = t^*$ can be zero.

for $i\epsilon\{1,2\}$. From (A.16) we know that $\partial g_i/\partial t_i > 0$, and $\partial x_i/\partial t_i < 0$ in the cooperative equilibrium. Therewith (A.23) and (A.24) imply that, while MRS remains unchanged, MRS$\,=1$, allowing capital mobility will increase the MRT. Having thus $\frac{\text{MRS}_i}{\text{MRT}_i}\Big|_{t^*,\text{comp}} < 1$, we know that a region could increase its utility from unilaterally reducing its tax level. We introduce $u_i(t_1,t_2) \equiv U(x_i(t_1,t_2), g_i(t_1,t_2))$ to express this point formally:

$$\exists t_1 < t^* \,\text{such that}\, u_1(t_1,t^*) > u_1(t^*,t^*). \tag{A.25}$$

As the cooperative equilibrium tax level t^* is by definition the tax which yields the maximally possible welfare for each country in a situation where both taxes are the same, we know in addition

$$u_1(t,t) < u_1(t^*,t^*) \,\forall t > t^*. \tag{A.26}$$

Finally, because for a given tax t_1, we know $\frac{\partial k_1}{\partial t_2} > 0$, as well as $\frac{\partial g_1}{\partial k_1} > 0$ and $\frac{\partial x_1}{\partial k_1} > 0$ when $t_2 \geq t_1$, we know that

$$t_2 > t^* \wedge t_2 \geq t \Rightarrow u_1(t,t_2) > u_1(t,t^*). \tag{A.27}$$

We thus see by contradiction, that no Nash equilibrium tax rates with $t_1^* = t_2^* > t^*$ can exist: for any such tax pair, we know that country 1 could make itself better of by unilaterally deviating to a value $t_i = t^*$: eqs. (A.25) and (A.26) and (A.27) imply that $\forall t > t^*$, $\exists t_1 \leq t^*$ such that $u_1(t_1,t) > u_1(t,t)$, which rules out any potential competitive equilibrium with tax rates $t_1 = t_2 > t^*$.As (A.25) rules out a Nash equilibrium at $t_1 = t_2 = t^*$ as well, we thus know that the equilibrium tax rates in the symmetric competitive situation are strictly below the optimal level:

$$t_{1,N}^* = t_{2,N}^* < t^*.$$

If we have a perfectly symmetric situation and thus a symmetric Nash equilibrium, it is trivial to see that the symmetric countries are worse off in the competitive situation: as the situation is perfectly symmetric, choosing an equilibrium tax $t_{1,N}^* = t_{2,N}^* < t^*$, yields for a country i the same utility it would have when instead being in autarky and choosing the same tax rate $t_{i,N}^*$ (the capital use $k_{i,N}^*$ remains anyway at k^* if the other country chooses the same tax rate t_N^*). Because for the autarkic country t^* is, however, the utility maximizing tax rate, the two competing countries using $t_{i,N}^* \neq t^*$ must have lower utilities than they would have in the cooperative equilibrium.

Finally, note that in this symmetric situation – despite their competitive behaviour – both countries choose the same tax rate in the equilibrium, and therefore at the equilibrium itself eq. (A.16) is valid. Because the taxes are here, however, lower in the two countries than in the cooperative equilibrium, while the capital employment is unchanged in both countries, the amount of public good provided is lower in the competitive equilibrium. ■ *Proposition I.1.*

A.2.1.2 Asymmetric Countries

Turning now to the asymmetric case where we may have $s_1 \neq s_2$. We start from the case where both have the same size, yielding the same equilibrium tax rate and capital employment in both countries. We will then examine what happens when the country sizes start to diverge.

Note that (A.2) implies $k_2 = (k^* - s_1 k_1)/s_2$. Together with (A.1) this implies that the arbitrage condition, (A.4), can be rewritten as:

$$f'(k_1) - t_1 = f'(\frac{k^* - s_1 k_1}{1 - s_1}) - t_2 = r. \tag{A.28}$$

Eq. (A.28) implies

$$(t_1 - t_2) = f'(k_1) - f'(\frac{k^* - s_1 k_1}{1 - s_1}). \tag{A.29}$$

For a given t_2 this implies (for $i \neq j$)

$$\frac{\partial k_i}{\partial t_i} = \frac{s_j}{s_j f''(k_i) + s_i f''(k_j)}, \tag{A.30}$$

or

$$\mathrm{d}f'(k_1) - \mathrm{d}f'(\frac{k^* - s_1 k_1}{1 - s_1}) = \mathrm{d}t_1. \tag{A.31}$$

Recalling that $f''(k_i) < 0$, thus see that for a given t_2 we have

$$\frac{\partial k_1}{\partial t_1} < 0. \tag{A.32}$$

Because of (A.32) we know that $\frac{-\mathrm{d}f'(\frac{k^* - s_1 k_1}{1 - s_1})}{\mathrm{d}t_1} > 0$, wherewith we know that (A.31) implies $\mathrm{d}f'(k_1) < \mathrm{d}t_1$, which, used in $f'(k_1) - t_1 = r$ from the arbitrage conditions (A.28) implies

$$\frac{\partial r}{\partial t_1} < 0.$$

Lemma 1

As the country sizes start to diverge from each other, the larger country can increase its utility if it increases its tax rate unilaterally, and the smaller country can increase its utility if it decreases its tax rate unilaterally, compared to the equilibrium taxes in the initial symmetric situation.

Proof of Lemma 1:

Assuming a regular, smooth enough production function, for $s_1 = s_2$ we have (starting) at the point $t_1 = t_2 = t_N$ (t_N is the Nash equilibrium tax rate for $s_1 = s_2$) and thus $k_1 = k_2 = k^*$ that

$$\begin{aligned} \mathrm{d}(t_1 - t_2) &= \mathrm{d}\left[f'(k_1) - f'(\frac{k^* - s_1 k_1}{1 - s_1})\right] = \mathrm{d}k_1 \cdot f''(k^*)\left[1 - (-\frac{s_1}{1 - s_1})\right] \\ &= \mathrm{d}k_1 \cdot f''(k^*)\left[\frac{1}{1 - s_1}\right], \end{aligned} \tag{A.33}$$

that is, starting at the point where $s_1 = s_2$ and $k_1 = k_2 = k^*$, for a marginally larger s_1, the reaction of domestic capital employment to a unilateral tax change, $\mathrm{d}k_1/\mathrm{d}t_1$, is (marginally) weaker [than the same reaction for $s_1 = 0.5$] (the reaction is in any case negative, as is easy to see), and vice versa.

Turning now to the reaction of the (net of tax) interest rate, recall the arbitrage condition $f'(k_1) - t_1 = r$ in (A.4). Using (A.33) in the arbitrage condition we have

$$\begin{aligned} \mathrm{d}r &= \mathrm{d}t_1\left[f''(k_1) \cdot \frac{1 - s_1}{f''(k^*)} - 1\right] \\ &= -s_1 \mathrm{d}t_1. \end{aligned} \tag{A.34}$$

Eq. (A.34) shows that, again starting at the point where $s_1 = s_2$ and $k_1 = k_2 = k^*$, for a marginally larger s_1, the reaction of the (net of tax) interest rate to a unilateral tax change,$\mathrm{d}r/\mathrm{d}t_1$, is (marginally) stronger [than at $s_1 = 0.5$] (it is in any case negative, as is easy to see), and vice versa.

Eqs. (A.33) and (A.34) show that, at $s_1 = s_2$ and $t_1 = t_2$ (and thus $k_1 = k_2 = k^*$) we have formally:

$$\partial \left[\left. \frac{\mathrm{d}r}{\mathrm{d}t_1} \right|_{t_i = t_\mathrm{N} \wedge s_1 = 0.5} \right] / \partial s_1 \;<\; 0 \tag{A.35}$$

$$\partial \left[\left. \frac{\mathrm{d}k_1}{\mathrm{d}t_1} \right|_{t_i = t_\mathrm{N} \wedge s_1 = 0.5} \right] / \partial s_1 \;>\; 0. \tag{A.36}$$

Note that relations (A.35) and (A.36) are both inversed if instead the *absolute* values of the inner terms on the LHS of the inequalities would be derived by s_1 instead of the real values, since both, the interest rate r as well as the domestic capital use, k_1, decrease as the tax rate t_1 is increased.

Consider now private consumption. In (A.9) we see that for the here considered case where $k^* = k_i$, the derivative of private consumption with respect to the tax choice is zero, as $\frac{\partial x_1}{\partial t_1} = -k^*$,which does not change for marginal changes in s_1. Thus, in the immediate vicinity of $s_1 = s_2 = 0.5$ and $t_1 = t_2 = t_\mathrm{N}$, the reaction of the domestic private consumption to a marginal unilateral domestic tax change is the same for $s_1 = 0.5$ as for a marginally larger or lower s_1.

[Note: The fact that the first term on the RHS of (A.8) increases (in non-absolute terms) with s_1 is intuitive: as the region becomes larger, lower production losses due to capital flight occur. The countering effect in the second term of the same equations RHS is also easily explained: a tax increase in a larger region depresses the global net-of-tax interest rate more strongly than a similar tax increase in a smaller region.]

For public consumption, it is straightforward to see in (A.10) that around $s_{1,\mathrm{ini}} = 0.5$ and $t_{1,\mathrm{ini}} = t_2$, where we have eq. (A.36), for a (marginally) larger s_1, the derivative $\frac{\partial g_1}{\partial t_1}$ becomes (marginally) larger than at $s_1 = 0.5$ itself. This result is intuitive as well: as the region becomes larger, capital can less easily flee the country if taxes increase, enhancing the effectiveness of the capital tax.

Taking all together, we thus see that in the immediate surrounding of $s_1 = s_2$ and $t_1 = t_2 = t_\mathrm{N}$, we have

$$\begin{aligned} \partial \left[\left. \frac{\partial x_1}{\partial t_1} \right|_{t_i = t_\mathrm{N} \wedge s_1 = 0.5} \right] / \partial s_1 &\;=\; 0 \\ \partial \left[\left. \frac{\partial g_1}{\partial t_1} \right|_{t_i = t_\mathrm{N} \wedge s_1 = 0.5} \right] / \partial s_1 &\;>\; 0. \end{aligned} \tag{A.37}$$

This shows that the larger country has an incentive to unilaterally increase the tax rate. By symmetry, we know that the smaller country improves its state when reducing its tax unilaterally. This is because, as (A.37) shows, the larger a country, the more easy it is to increase the provision of the public good by an increase of the domestic tax rate, and vice versa. Defining $t_i^R(t_j)$ as the optimal unilateral tax level (reaction) chosen by country i if the other country chooses the tax level t_j, we thus have

$$\frac{\partial t_1^R(t_2 = t_\mathrm{N})}{\partial s_1} > 0, \; \frac{\partial t_2^R(t_1 = t_\mathrm{N})}{\partial s_1} < 0.$$

■ *Lemma 1.*

Lemma 2

If $t_i^R(t_j)$ is country i's optimal reaction tax to the competing country j's tax choice, and t_i^{fix} is the tax rate for which the reaction maps the argument onto itself, $t_i^{fix} = \{t_i | t_i = t_i^R(t_i)\}$, we have that $\frac{\partial t_i^R(t_i^{fix})}{\partial s_j} > 0$.

Proof of Lemma 2:

Without loss of generality we proof the claim for $i = 1$. To emphasize the analogy to the proof of Lemma 1, we define $t'_N \equiv f_1^{fix}$.

Assuming a regular, smooth enough production function, we have (starting) at the point $t_1 = t_2 = t'_N = f_1^{fix}$ and thus $k_1 = k_2 = k^*$ that

$$\begin{aligned} \mathrm{d}(t_1 - t_2) &= \mathrm{d}\left[f'(k_1) - f'(\frac{k^* - s_1 k_1}{1 - s_1})\right] = \mathrm{d}k_1 \cdot f''(k^*)\left[1 - (-\frac{s_1}{1 - s_1})\right] \\ &= \mathrm{d}k_1 \cdot f''(k^*)\left[\frac{1}{1 - s_1}\right], \end{aligned} \tag{A.38}$$

that is, for any s_1, starting at the point where $k_1 = k_2 = k^*$, for a marginally larger s_1, the reaction of domestic capital employment to a unilateral tax change, $\mathrm{d}k_1/\mathrm{d}t_1$, is (marginally) weaker (the reaction is in any case negative, as is easy to see), and vice versa.

Turning now to the reaction of the (net of tax) interest rate, recall the arbitrage condition $f'(k_1) - t_1 = r$ in (A.4). Using (A.38) in the arbitrage condition we have

$$\begin{aligned} \mathrm{d}r &= \mathrm{d}t_1\left[f''(k_1) \cdot \frac{1 - s_1}{f''(k^*)} - 1\right] \\ &= -s_1 \mathrm{d}t_1. \end{aligned} \tag{A.39}$$

Eq. (A.39) shows that, again starting at the point where $k_1 = k_2 = k^*$, for a marginally larger s_1, the reaction of the (net of tax) interest rate to a unilateral tax change,$\mathrm{d}r/\mathrm{d}t_1$, is (marginally) stronger (it is in any case negative, as is easy to see), and vice versa.

Eqs. (A.38) and (A.39) show that, at $t_1 = t_2$ (and thus $k_1 = k_2 = k^*$) we have formally:

$$\partial\left[\left.\frac{\mathrm{d}r}{\mathrm{d}t_1}\right|_{t_i = t'_{\mathrm{N}}}\right]/\partial s_1 < 0 \tag{A.40}$$

$$\partial\left[\left.\frac{\mathrm{d}k_1}{\mathrm{d}t_1}\right|_{t_i = t'_{\mathrm{N}}}\right]/\partial s_1 > 0, \tag{A.41}$$

Note that relations (A.40) and (A.41) are both inversed if instead the *absolute* values of the inner terms on the LHS of the inequalities would be derived by s_1 instead of the real values, since both, the interest rate r as well as the domestic capital use, k_1, decrease as the tax rate t_1 is increased.

Consider now private consumption. In (A.9) we see that for the here considered case where $k^* = k_i$, the derivative of private consumption with respect to the tax choice is zero, as $\frac{\partial x_1}{\partial t_1} = -k^*$,which does not change for marginal changes in s_1. Thus, in the immediate vicinity of $t_i = t'_N$ (implying $k_i = k^*$), the reaction of the domestic private consumption to a marginal unilateral domestic tax change does not change with a marginal reduction of s_1.

[Note: The fact that the first term on the RHS of (A.8) increases (in non-absolute terms) with s_1 is intuitive: as the region becomes larger, lower production losses due to capital flight occur. The countering effect in the second term of the same equations RHS is also easily explained: a tax increase in a larger region depresses the global net-of-tax interest rate more strongly than a similar tax increase in a smaller region.]

For public consumption, it is straightforward to see in (A.10) that around $t_{1,\text{ini}} = t_2$, where we have eq. (A.41), for a marginally increasing s_1, the derivative $\frac{\partial g_1}{\partial t_1}$ increases (marginally) as well. This result is intuitive as well: as the region becomes larger, capital can less easily flee the country if taxes increase, enhancing the effectiveness of the capital tax.

Taking all together, we thus see that in the immediate surrounding of $t_i = t'_{\text{N}}$ (and thus $k_1 = k_2 = k^*$), we have

$$\begin{aligned} \partial \left[\left. \frac{\partial x_1}{\partial t_1} \right|_{t_i = t'_{\text{N}}} \right] / \partial s_1 &= 0 \\ \partial \left[\left. \frac{\partial g_1}{\partial t_1} \right|_{t_i = t'_{\text{N}}} \right] / \partial s_1 &> 0. \end{aligned} \tag{A.42}$$

This shows that the larger country has an incentive to unilaterally increase the tax rate, that is we have

$$\frac{\partial t_1^R(t_2 = t'_{\text{N}})}{\partial s_1} > 0.$$

■ *Lemma 2.*

Lemma 3

If $t_i^R(t_j)$ is country i's optimal reaction tax to country j's tax choice, we have $\frac{\partial t_i^R(t_j)}{\partial t_j} < 1 \forall t_j$.

Proof of Lemma 3:

Consider a given country size relationship (s_1, s_2), and write $t_1^R(t_2)$ the optimal tax choice t_1 of country 1 as a function of a hypothetical tax choice t_2 of country 2. Consider a 'starting' (index s) point for the taxes, (t_1^s, t_2^s), for which t_1^s is the optimal reaction to country 2's tax choice: $t_1^s = t_1^R(t_2^s)$. For this to hold, we must have, for country 1, $\text{MRS}_{1,s} = \text{MRT}_{1,s}$. Consider now a marginal one-per-one increase of both taxes, leading to the 'end' (index e) point with taxes $t_i^e = t_i^s + \varepsilon$. We know that this change does neither affect the level of k_i, nor its derivative $\partial k_i / \partial t_i$ in either country (see (A.29) and (A.30)). At the same time, the increase in the domestic tax itself has reduced $\partial g_1 / \partial t_1$ (see (A.10)),[2] but left $\partial x_i / \partial t_i$ unchanged (see (A.9)), that is $\text{MRT}_{1,e} > \text{MRT}_{1,s}$. In parallel, because k_i and $\partial k_i / \partial t_i$ have remained constant along the shift from t_i^s to t_i^e, we know that $x_{1,e} < x_{1,s}$ (see (A.6)) and $g_{1,e} > g_{1,s}$ (see (A.7)), implying $\text{MRS}_{1,e} < \text{MRS}_{1,s}$. We thus necessarily have an overprovision of the public good at the 'end' state, $\text{MRS}_{1,e} < \text{MRT}_{1,e}$. Thus, we find $t_{1,e} \neq t_1^R(t_{2,e})$, and therewith know that the derivative of the reaction function is never unity:

$$\frac{\partial t_i^R(t_j)}{\partial t_j} \neq 1. \tag{A.43}$$

Ruling out the case where the domestic tax grows arbitrarily large as a reaction to the other country setting a very large tax rate,[3] and assuming the problem to be well-behaved enough for the tax reaction curves to be smooth (that is, assuming their first derivative not to exhibit any jumps in the interior), (A.43) implies

$$\frac{\partial t_i^R(t_j)}{\partial t_j} < 1. \tag{A.44}$$

[2] Note that $\partial g_1 / \partial t_1$ is positive (in the region of the optimal tax reaction by country 1 we must always have $\partial g_1 / \partial t_1 > 0$, as else $MRS_1 = MRT_1$ could not be fulfilled since we always have $\partial x_i / \partial t_i > 0$).

[3] This assumption makes intuitive sense: even if the competing country sets a tax rate infinitely large, domestic capital remains finite and taxing it at a very large rate would necessarily drive private consumption down to zero, a case which is ruled out by the model-assumption of the marginal utility becoming infinitely large as consumption of one of the two goods approaches zero. See also Wilson (1991) arguing in this direction.

■ *Lemma 3.*

Note that this result must hold in any case where we can truly talk of a tax-competition where both countries impose negative externalities against each other by setting taxes lower to attract additional capital: to see this, consider a case in which we would have $\partial t_1^R(t_2)/\partial t_2 > 1$. In this case country 2 would know that when it marginally increases its tax rate, country 1 would increase its own tax rate even more, which implies that in this case rather than losing capital by increasing its domestic tax rate, country 2 could attract capital by increasing its own tax rate! This would also imply that country 2 could set its tax rate larger than socially optimal, overproviding rather than underproviding its public good, a situation which hardly corresponds to the natural understanding of a situation with tax competition.[4]

Proposition I.2

In a Nash equilibrium, the smaller country sets the lower tax.

Proof of Proposition I.2:

Given Lemmata 2, $\frac{\partial t_i^R(t_i^{fix})}{\partial s_j} > 0$, and 3, $\frac{\partial t_i^R(t_j)}{\partial t_j} < 1$, it is easy to see that in a Nash equilibrium with $s_i > s_j$ we necessarily have $t_i > t_j$. Note first that a Nash equilibrium can be defined as the set of tax rates t_i for which we have $t_1 = t_1^R(t_2) \wedge t_2 = t_2^R(t_1)$. Because the reaction functions are strictly positive functions,[5] i.e., $t_i^R(t_j) > 0 \forall t_j$, this condition can only be fulfilled if

$$\exists i \neq j \,\text{such that}\, t_j \geq t_i^{fix} \wedge t_i \leq t_j^{fix}. \tag{A.45}$$

Lemma 2 together with Lemma 3 implies that

$$s_i > s_j \Rightarrow t_i^{fix} > t_j^{fix}, \tag{A.46}$$

which is established as follows: Lemma 3 indicates that $t_i^R(t_j)$ has a slope of less than 1. Be t_o the t_1^{fix} for a specific relative country size $s_1 = s_o$, and Δ the increase of t_1^{fix} when the country size s_1 is increased marginally by ε to $s_1 = s_o + \varepsilon$. We have $\Delta = \Delta \,\text{such that}\, t_o + \Delta = t_1^R(t_o + \Delta)|_{s_1}$, which is approximated by $t_o + \Delta = t_1^R(t_o)|_{s_o} + \varepsilon \frac{\partial t_1^R(t_o)}{\partial s_1} + \Delta \frac{\partial t_1^R(t_o)}{\partial t_2}$, implying, because of $t_1^R(t_o)|_{s_o} = t_o$, that $\frac{\Delta}{\varepsilon} = \frac{\partial t_1^R(t_o)}{\partial s_1} / (1 - \frac{\partial t_1^R(t_o)}{\partial t_2})$, which we know to be positive by Lemmata 2 and 3, confirming (A.46).

It is easy to see with (A.45) and (A.46) that given that the slopes of the reaction functions are smaller than 1, a Nash equilibrium can only exist with $s_i > s_j \Rightarrow t_i > t_j$. See also a graphical illustration for a similar argument in Wilson (1991). ■ *Proposition I.2.*

Proposition I.3

If preferences are separable in public and private consumption, we have that, as the size of the small country becomes small enough, its competitively chosen Nash equilibrium tax rate is lower, and its utility is larger, than in the cooperative equilibrium. As the size of the small country becomes smaller and smaller, the tax rate and the utility of the large country

[4] Note that while (A.44) is not directly relevant to the issue of the stability of a Nash equilibrium (convergence of the tax rates in the immediate surrounding of the equilibrium point would be guaranteed by $\left| \frac{\partial t_i^R(t_j)}{\partial t_j} \frac{\partial t_j^R(t_i)}{\partial t_i} \right| < 1$ instead, for which (A.44) is neither necessary nor sufficient), it does guarantee stability and uniqueness of a Nash equilibrium if we in addition require $\frac{\partial t_i^R(t_j)}{\partial t_j} > -1$, an assumption which intuitively seems to make sense, as it would be rather counterintuitive if the best reaction to a foreign country reducing the tax rate by a certain value would be to reduce the own tax rate by even more than that value.

[5] Given (A.13) a country is always strictly better off setting a positive tax rate.

become arbitrarily close to their values of the cooperative equilibrium but remain strictly below.

Proof of Proposition I.3.

We designate with a "*" the values of the variables when both countries choose the tax rates that correspond to those in the cooperative equilibrium, that is, besides t^* we now write x^*, g^*, MRS*, MRT$_i^*$, U_x^*, U_g^*, where we omit the country index in those cases where it is obvious that the values are the same for both countries in the cooperative equilibrium. We call Δ_i the deviation of the Nash equilibrium tax rates from their counterpart from the cooperative equilibrium, so that if $t_{i,N}$ is the Nash equilibrium tax rate, we have $\Delta_i \equiv t_{i,N} - t^*$.

Country 1 is the large country. According to the claim in the Proposition we assume $\lim_{s_2\to 0} \Delta_1 = 0^-$, that is, that Δ_1 approaches zero but remains strictly negative. Further, again according to what is claimed, Δ_2 is supposed to be negative (more than marginally), but it is of course finite since no negative total taxes can be imposed in any country.

Assuming preferences to be separable, a first order Taylor expansion of the marginal utilities in the first order condition for (the large) country 1 yields

$$(U_x^* + U_{xx}^* \sum_i [\Delta_i \frac{\partial x_1^*}{\partial t_i}]) \left. \frac{\partial x_1}{\partial t} \right|_N + (U_g^* + U_{gg}^* \sum_i [\Delta_i \frac{\partial g_1^*}{\partial t_i}]) \left. \frac{\partial g_1}{\partial t} \right|_N = 0,$$

where derivatives to be evaluated at the Nash equilibrium values are indexed by an N.[6]

Having $\frac{\partial k_i}{\partial t_i} = \frac{s_j}{s_j f''(k_i) + s_i f''(k_j)}$, see that $\lim_{s_2\to 0} \frac{s_2}{f''(k_2)} = 0$, which simplifies the relevant derivatives. Those for the case of the tax rates from the cooperative equilibrium become

$$\begin{array}{cc} \frac{\partial x_1^*}{\partial t_1} = -k^* & \frac{\partial x_1^*}{\partial t_2} = 0 \\ \frac{\partial g_1^*}{\partial t_1} = k^* + t^* \frac{s_2}{f''(k_2)} & \frac{\partial g_1^*}{\partial t_2} = -t^* \frac{s_2}{f''(k_2)}, \end{array}$$

and those evaluated at the Nash equilibrium values are

$$\begin{aligned} \frac{\partial x_1}{t_1} &= [k^* - k_1] f''(k_1) \frac{s_2}{f''(k_2)} - k^* \\ \frac{\partial g_1}{t_1} &= k_1 + t_{1,N} \frac{s_2}{f''(k_2)}. \end{aligned}$$

The expanded FOC thus writes

$$\begin{aligned} &(U_x^* + U_{xx}^*(-\Delta_1 k^* + 0))([k^* - k_1] f''(k_1) \frac{s_2}{f''(k_2)} - k^*) \\ &+ (U_g^* + U_{gg}^*(\Delta_1(k^* + t^* \frac{s_2}{f''(k_2)}) + \Delta_2(-t^* \frac{s_2}{f''(k_2)})))[k_1 + t_{1,N} \frac{s_2}{f''(k_2)}] \quad = 0. \end{aligned}$$

To simplify this term, we will often apply the rule that marginally low variables multiplied with other marginally low variables become negligibly low and can be ignored. That is, e.g., $s_2[k^* - k_1]$ will be ignored because it converges to zero with twice the 'speed' of simple variables here converging to zero, and $\Delta_1 k_1$ will be considered as $\Delta_1 k^*$ because the marginal difference

[6]Note that, while Δ_2 is itself not marginally small, the implied changes in x_1 and g_1 are still only marginal (because due to the converging size of $s_2 \to 0$, the tax rate of country 2 has a vanishing influence on the large country).

between k_1 and k^*, multiplied by the marginal Δ_1 converges with double-speed as well. We thus get

$$-U_x^* k^* + k^{*2}\Delta_1 U_{xx} + U_g^* \left[k^* + \frac{s_2}{f''(k_2)}[t^* + 2\Delta_1 - \Delta_2]\right] + U_{gg}k^*(\Delta_1 - \frac{t^*\Delta_2 s_2}{f''(k_2)}) = 0, \quad \text{(A.47)}$$

and solved for Δ_1:

$$\Delta_1 = \frac{s_2}{f''(k_2)} \frac{U_g^*(\Delta_2 - t^*) + U_{gg}k^* t^* \Delta_2}{k^*(k^* U_{xx} + U_{gg})}, \quad \text{(A.48)}$$

which confirms that there exists a Δ_1 which, as s_2 approaches zero, approaches zero as well. While Δ_2 (and thus t_2) influences the exact values of Δ_1, the convergence takes place for any sensible value that t_2 may take on. We thus have

$$\lim_{s_2 \to 0^+} \Delta_1 = 0, \quad \text{(A.49)}$$

or, equivalently, $\lim_{s_2 \to 0^+} t_{1,N} = t^*$. We use this preliminary result to approximate the capital employed in country 2 as a function of t_2. Benefitting from the convergence of $t_{1,N}$ and recalling that this implies $\lim_{s_2 \to 0} k_1 = k^*$, the arbitrage equation, (A.4), yields $k_2 = f'^{-1}(t_{2,N} - t_{1,N} + f'(k^*))$, and we thus have $\partial k_2/\partial t_2 = [f''(t_{2,N} - t_{1,N} + f'(k^*))]^{-1} < 0$.

With $t_{1,N}$ arbitrarily close to t^*, we know that $\text{MRS}_{t^*,2} \approx 1$, that is, if country 2 chooses $t_2 = t^*$, its consumption bundle is arbitrarily close to that of the cooperative equilibrium, implying also an arbitrarily close marginal rate of substitution.[7] How about the corresponding marginal rate of transformation, $\text{MRT}_{t^*,2}$? For $t_2 = t^*$ and thus $\lim_{s_2 \to 0} k_2 = k^*$ we have $\partial x_2/\partial t_2 = -k^*$ and $\partial g_2/\partial t_2 = k^* + t_2 \frac{\partial k_2}{\partial t_2}$, that is, we have $\text{MRT}_{t^*,2} = \left.\frac{-\partial x_2/\partial t_2}{\partial g_2/\partial t_2}\right|_{t_2 = t^*} > 1$. We therewith know $\text{MRS}_{t^*,2} < \text{MRT}_{t^*,2}$, i.e., were country 2 to stick to $t_2 = t^*$, it would – in terms of foregone private consumption – pay an inefficiently high price for the marginal public goods provided, relative to the marginal utilities it derives from the two goods. Accordingly, reducing t_2 increases the domestic utility. Assuming a minimal amount of public spending to be essential, $\text{MRS}_{t_2,2}$ will equate $\text{MRT}_{t_2,2}$ for a specific $t_2 < t^*$. This is the optimal Nash equilibrium tax rate for country 2 – using a lower rate is not desirable either, because there $\text{MRS}_{t_2,2} > \text{MRT}_{t_2,2}$.

Now that we have seen that we have $t_{2,N} < t^*$ in equilibrium, i.e., we have $\Delta_2 < 0$, we see in (A.48) that Δ_1 is strictly lower than zero: the first fraction on the RHS is smaller than 0, the nominator of the second fraction as well, and also its denominator. We thus have confirmed that the equilibrium level of the tax of the large country is (marginally) lower than in the cooperative equilibrium. We omit formal analysis confirming that no other equilibrium is possible. We see that for country 1 to deviate more than marginally would imply utility losses independently of t_2. For country 2 in turn its tax choice is only marginally influenced by changes in t_1. Thus the characterized tax choices seem not to be only two mutually optimal tax choices among many, but instead they should represent the only interior Nash-equilibrium.

Finally, it is easy to understand that the utility level of the small country is not only above that of the large country but also above the utility of the countries in the cooperative equilibrium: the marginal change in the large country's tax rate by Δ_1 can only marginally affect the utility level of country 2 when its own tax choice remains at $t_2 = t^*$. From that point on, however, country 2's utility increases more than marginally when its tax rate changes from t^* to the optimal level, such that overall a utility level higher than in the cooperative situation is achieved in country 2. Country 1 suffers a utility loss compared to the cooperative equilibrium, which is most easily seen by considering that country 2 attracts part of country 1's endowment,

[7] Note, because in the cooperative economy, we had MRT = 1, as capital was fixed and taxes thus have the same effect as lump sum transfers without any losses.

wherewith country 1's utility must necessarily decrease compared to the cooperative state. ■ *Proposition I.3.*

A.2.2 Leviathan

Consider the symmetric case ($s_i = 0.5$) and a Leviathan state, that is, a government which, instead of caring for the utility of its citizens, maximizes the state revenue. In this case we have:

Proposition II

(i) *Absence of tax competition.* If (*situation a*) the only limiting upper bound on the tax rate is that the residents must be left with nonnegative private net income, the Leviathan will set tax rates high enough to confiscate all private revenues. If (*situation b*) the Leviathan cannot tax capital by more than its marginal productivity, the Leviathan will tax the capital at its marginal productivity (that is, it chooses the maximally allowable tax), such as to just confiscate all capital interest revenues. This leaves private labour income to the workers. Public good is overprovided in both situations.

(ii) *With tax competition.* Competition *may*[8] induce the Leviathan to lower the tax compared to an autarkic Leviathan state, but the tax chosen will necessarily be above the competitive benevolent's choice. In this case, the tax competition may either increase the utility of the region compared to the autarky-case (if the Leviathan was not able to set too high taxes in any case and if the competition lowers the tax just somewhat below the optimal level) or it may reduce the tax so far as to even decrease the overall utility compared to the Leviathan. Furthermore, the utility under a competitive Leviathan can also be larger or lower than under a competitive benevolent government, depending on the functional forms of the utility and the production function.

Proof Proposition II:

(i) *Absence of tax competition*: capital can then be considered fixed, $k_i = k^*$. In this case $g_i = t_i k^*$, and $x_i = f(k^*) - t_i k^*$.

The FOC is $\partial g/\partial t \overset{!}{\geq} 0 \perp t = \bar{t}$, where $\bar{t}$ is the external upper limit on the tax rate that can be imposed, e.g., because the tax cannot exceed the marginal revenue of capital or because the government cannot apply a tax that yields negative net revenues for its population. Because of $\partial g/\partial t = k^*$ under cooperation (or autarky), no interior solution can exist, that is, we have $t = \bar{t}$. This may be the average productivity of capital, $t = f(k^*)/k^*$, in which case private consumption x_i collapses to zero. If instead only capital revenues can be extracted from the population, that is, we have $t = f'(k)$, private net revenues will be positive, as, due to decreasing capital productivity, we have $x_i = f(k^*) - f'(k^*)k^* > 0$. Despite the positive private net revenue left to the population, under the assumption that preferences and production possibilities are such that a utility (that of its population) maximizing government would generally imply taxes that leave positive capital revenues, it is straightforward to understand that the tax rate is also in this case suboptimally high and utility could be increased by setting lower tax rates.

(ii) *With tax competition*: In the symmetric equilibrium we will have $k_i = k^*$, but capital *per se* is mobile, and the arbitrage condition implies $\partial k_i/\partial t_i = [2f''(k^*)]^{-1}$. We thus have $g_i = t_i k_i$,

[8] Depending on the functional forms of $f(\cdot)$ and $U(\cdot, \cdot)$, if an interior equilibrium exists, the Leviathan may alternatively just behave as in (i) (assumed that tax rate restrictions exist), if the the production function is too strongly curved at k^*. The intuition behind this is as follows: because with a strong enough curvature of the production function $f(k)$, a region can hardly attract capital from the other region by lowering taxes, since the marginal productivity decreases so rapidly as capital moves from one region to the other.

and the Leviathan's optimality condition, $\partial g_i/\partial t_i \stackrel{!}{=} 0$, here becomes $k_i \stackrel{!}{=} -t_i\frac{\partial k_i}{\partial t_i}$, yielding

$$t_L = -2k^* f''(k^*), \tag{A.50}$$

where t_L is the tax rate that the Leviathan governments of both symmetric countries choose in the Nash competition. For a large enough curvature $|f''(k^*)|$, condition (A.50) implies that the optimal tax rate – assuming the Nash equilibrium to exist – is again very high and eventually restricted not by the tax competition but by fundamental bounds such as $r \geq 0$ or $x \geq 0$, with the outcome thus the same as in case (i). But if $|f''(k^*)|$ is low enough, the tax competition affects the choice of the tax rate, and the following examines this case. Obviously, for a small enough (in absolute terms) curvature, at least for $\lim f''(k^*) \to 0$, it is straightforward to see that the optimality condition, (A.50), implies a very low tax rate. However, as a comparison to the symmetric case with a *competitive benevolent* (in terms of maximizing *domestic* utility in a country) government shows, the Leviathan's tax choice is always above the competitive benevolent's tax rate: in the case of the competitive benevolent government (with $s_i = 0.5$) we have the FOC $U_g(k_i + t_i \cdot [2f''(k^*)]^{-1}) + U_x(-k^*) \stackrel{!}{=} 0$, yielding $t_N = -2k^* f''(k^*)[1 - \frac{U_x}{U_g}]$, which is necessarily smaller than t_L (note that in the competitive equilibrium we always have $U_x/U_g < 1$).

Is the tax in the competitive Leviathan case above or below the socially optimal level? To answer this question, see that, depending on the functions $U_x(x, g)$ and $U_g(x, g)$, but independently of the exact nature of the production function $f(k)$,[9] the socially optimal tax rate t^* can take on any value strictly between 0 and $\bar{t}$,[10] where $\bar{t}$ is the exogenously given, finite upper bound on the allowable tax rate. Because t_L is independent of preferences (that is, independent of the shape of U), and because, depending on the shape of $f(k)$, it can take on any value between 0 and $\bar{t}$, we know that in general, both cases, of t_L falling short of or exceeding t^*, are possible.

Regarding the utilities, it is clear that the argument just explained also implies that introducing the tax competition among Leviathan states can both increase or lower the populations' utilities, depending on the restrictions on the maximally possible tax level in autarky (or cooperation) and on the functional forms of the utility and the production. For a similar argument, in the possible case where t_L exceeds the socially optimal tax t^*, both cases, of $U|_{t_L}$ being above or below the utility under *a competitive* benevolent government, $U|_{t_N}$, are in general possible: to understand this, see first that in the case where we have $t_L < t^*$, the utility under the Leviathan must necessarily be larger than that under the (also competitive) benevolent, as the latter sets an even lower tax than the Leviathan. Second, for the other case, that is when we have $t_L > t^*$, consider Figure A.1: while t_L is independent of the utility function, we know that $t_N = t_L \underbrace{[1 - \frac{U_x}{U_g}|_N]}_{<1}$. As the diagram for the case when t_L exceeds the optimal level t^* illustrates, depending on the form of the marginal utility curves, the utility loss (written ΔU and defined as the utility in the social optimum minus the achieved utility) from the Leviathan tax, ΔU_L, can exceed that from a benevolent competitive governance (ΔU_N) as in the drawn example, but with alternative curves we could also find the inverse. Note that, as g and x increase resp. decrease linearly in the tax rate, the surfaces in the diagram are proportional to the utility losses. ■ *Proposition II.*

[9]The assumptions of finite derivatives of the production function, as described in the model definition, are assumed to be respected.

[10]Consider the functional properties imposed with the model assumptions.

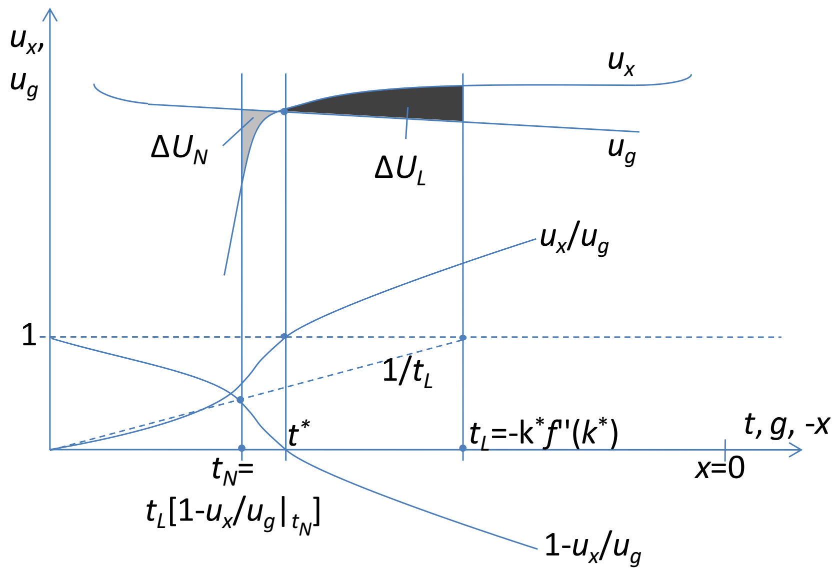

Figure A.1: Utility losses with taxes by Leviathan (t_L) and benevolent competitive government (t_N), when t_L exceeds socially optimal value t^*.

A.2.3 Stackelberg

Stackelberg Leader-Follower-scheme definition

Country 2 is the *Stackelberg follower*, that is it acts in the exact same way we assumed all actors to act in the competitive Nash setting: it takes the other country's tax choice as given and reacts to it in the way which yields the highest utility under the assumption that the competing country 1 does not deviate from its initial tax choice.

Country 1 is the *Stackelberg leader*, whose behaviour differs from that of a Nash actor: as the leader, country 1 acts strategically, anticipating the implications of its own tax choice on the tax choice of the competing country 2.

We thus have for country 2 still the usual reaction function of an unstrategically competing country, characterised by the first-order condition

$$U_x \underbrace{\left([k^* - k_2] f''(k_2) \frac{\partial k_2}{\partial t_2} - k^* \right)}_{<0} + U_g \left(k_2 + t_2 \frac{\partial k_2}{\partial t_2} \right) = 0,$$

where $\frac{\partial k_2}{\partial t_2} = \frac{s_1}{s_1 f''(k_2) + s_2 f''(k_1)}$.

Country 1 as the leader maximizes its utility subject to the usual Nash equilibrium constraints, but considering the reaction function of its competitor, $t_2^R(t_1)$, instead of assuming t_2 as exogenously given. Because the tax choice of the follower affects the utility of the leader only through the capital channel, and the capital shares depend – besides on the size ratios – only on the *difference* between the taxes in the two countries, the sole difference between the Nash strategies and the Stackelberg strategies can readily be accounted for by adapting the formula for the reaction of the domestic capital intensity to the Stackelberg leader's (index SL) tax choice:

$$\left.\frac{\partial k_1}{\partial t_1}\right|_{SL} = \frac{s_2}{s_2 f''(k_1) + s_1 f''(k_2)} (1 - \frac{\partial t_2^R(t_1)}{\partial t_1}).$$

Proposition III:

If both countries compete in a way such that each country's optimal reaction to a tax rate increase by the other country is an increase of the domestic tax rate, the tax rates in the Stackelberg equilibrium exceed those in the competitive Nash equilibrium. The tax rate of the Stackelberg leader rises more strongly, but both countries are better off in the Stackelberg equilibrium. Which country has the larger utility depends notably on the size ratio: on one hand, if both have the same size, the follower is better off than the leader. On the other hand, if the size of the Stackelberg leader is small enough, it sets the lower tax and enjoys larger utility than the follower.

Proof of Proposition III:
Restricting the attention to the case where the reaction function has a positive slope, that is, assuming that the unstrategic equilibrium response of a country to a unilateral tax increase by its foreign competitor is to increase the domestic tax rate as well, we show that the Stackelberg equilibrium is characterized by taxes that are above those of the competitive Nash equilibrium in both countries, and that the taxes of the Stackelberg leader rise more strongly than those of the follower. Both countries are better off in the Stackelberg equilibrium. For the symmetric equilibrium we have the somewhat counterintuitive situation that the follower is better off than the leader, even though the latter behaves 'smarter', by anticipating the reactions of his competitor. This is reversed if the size of the leading country becomes small enough. In this case, the Stackelberg leader sets the tax lower than the follower, and the leader also receives the larger utility.

First, consider the Nash equilibrium, with the tax rates $t_{i,N}$, in which both countries find themselves if they disregard the effect of their own tax choices on their competitor's tax choice. We know that in the Nash equilibrium, the countries are necessarily in a situation where the relative marginal value of public good is larger than one, $\text{MRS} \equiv \frac{U_g}{U_x} > 1$, as considered unilateral increases of the domestic tax imply, besides a transfer from private to public good, the loss of parts of the overall consumption through the tax-induced outflow of capital. Thus, the sensitivity of the amount of capital employed domestically to the own tax changes limits the chosen tax level. Consider now, how the strategic foresight of country 1 changes its perception of the situation when he changes from a 'Nash actor' to a Stackelberg leader. Similarly to the case where he was acting non-strategically, he considers potential changes in the domestic capital intensity that his choice implies. But he now recognizes that the capital response to his domestic tax change are overall less strong than his earlier 'Nash' view had suggested: anticipating now that country 2 will follow country 1's tax change by a change of t_2 into the same direction (recall $0 < \partial t_2^R/\partial t_1 < 1$), the perceived negative response of k_1 to an increase in t_1 is attenuated, i.e., the marginal rate of transformation in the Nash equilibrium state, is larger for the Stackelberg leader (index SL) than that which a 'Nash actor' (index N) would have perceived in that equilibrium. We write $\text{MRT}_{1,SL}|_{t_{i,N}} > \text{MRT}_{1,N}|_{t_{i,N}}$. Since the initial Nash equilibrium was characterized by the marginal rate of transformation equalling the marginal rate of substitution in both countries, $\text{MRS}_{p,N}|_{t_{i,N}} = \text{MRT}_{p,N}|_{t_{i,N}}$ for $p = \{1, 2\}$, we know that we have $\text{MRT}_{1,SL}|_{t_{i,N}} > \text{MRS}_{1,N}|_{t_{i,N}}$, and the country will increase its tax rate until the 'strategic' marginal rate of substitution equates the marginal rate of transformation. Because the reaction function of country 2 has a positive slope, we know that the resulting Stackelberg equilibrium tax rate of country 2 exceeds its Nash equilibrium tax rate as well. Furthermore, because $\partial t_2^R/\partial t_1 < 1$, we know that the tax rate in the follower country 2 raises less than that in the leading country 1. We thus conclude that

$$\begin{aligned} t_{i,S} &> t_{i,N} \\ t_{2,S} - t_{2,N} &> t_{1,S} - t_{1,N}. \end{aligned}$$

It is easy to see that both countries are better off in the Stackelberg equilibrium than in the competitive Nash equilibrium: If the leading country would have been better off if in the Nash equilibrium it would not have strategically chosen to deviate from that Nash equilibrium. The fact that the leading country increases its tax in the Stackelberg equilibrium compared to its Nash choice, implies that the follower country would have a larger utility in the Stackelberg situation, if it were to stick to its Nash tax rate, than in the original Nash equilibrium. Further, the fact that the country decides to choose a different tax rate in the Stackelberg equilibrium shows that it is even able to reap a 'second' utility increase (otherwise it would still choose the Nash tax rate); thus also the follower country is better off in the Stackelberg equilibrium than in the Nash equilibrium.

In the case where the two countries have the same size and are initially in a symmetric Nash equilibrium, the fact that $t_{2,S} < t_{1,S}$ implies that country 2 must be better off in the Stackelberg equilibrium than the leading country 1: if country 2 would unilaterally change its tax rate to the larger $t_{1,S}$. The therewith achieved utility of country 2, call it U_2', would exceed the utility of country 1 in the Stackelberg equilibrium, $U_2' > U_{1,S}$, because after that hypothetical move both countries would find themselves in the same state as country 1 in the Stackelberg equilibrium, except for the capital intensity, which would, with $k_i' = k^*$, be larger, since in the Stackelberg equilibrium we have $k_{1,S} < k^*$ because country 2 had attracted foreign capital through its lower tax rate, $t_{2,S} < t_{1,S}$. With the larger capital stock the two countries would, after the described hypothetical move, also enjoy a larger utility than country 1 has in the Stackelberg equilibrium. The fact that country 2 does in reality not unilaterally deviate from $t_{2,S}$ to the larger tax rate $t_{1,S}$, even though it would there enjoy an even higher utility than country 1 does in the Stackelberg equilibrium, implies that we must have

$$U_{2,S} > U_{1,S}.$$

The same does not hold true for all situations with unequal country sizes, that is, there exist situations in which the Stackelberg leader is better off than the follower. We are going to see this for a leading country whose size approaches zero, $\lim s_1 \to 0$. In this case, as has been shown in Proposition I.3 (see eqs. A.48 and A.49), the influence of t_1 on the tax of the unstrategically reacting country, here $t_{2,S}$, is vanishingly small even for large changes in the foreign tax rate t_1, that is, we have $\lim_{s_1\to 0} \partial t_2/\partial t_1 = 0$. Therewith, the capital reaction function considered by the leading country collapses in the limit to that of a nonstrategic Nash actor of marginal size:

$$\lim_{s_1\to 0} \frac{\partial k_1}{\partial t_1} = \lim_{s_1\to 0} \frac{s_2}{s_2 f''(k_1) + s_1 f''(k_2)}(1 - \underbrace{\frac{\partial t_2}{\partial t_1}}_{=0}) = \frac{1}{f''(k_1)}.$$

The optimization problem for a strategic country of a very small size thus approaches that of a non-strategic country of the same size. Therewith, their optimal tax rates are also the same asymptotically, $\lim_{s_1\to 0} t_{1,S} = \lim_{s_1\to 0} t_{1,N}$. In this case the large country, which is the follower, also sets the same tax rate in the Stackelberg competition as it would in the Nash equilibrium. From Proposition I.3 we know thus that in this case, the small Stackelberg leader sets the lower tax than the large follower, $\lim_{s_1\to 0} t_{1,S} < \lim_{s_1\to 0} t_{2,S}$, and has a larger utility, $\lim_{s_1\to 0} U_{1,S} > \lim_{s_1\to 0} U_{2,S}$. ■ *Proposition III.*

A.2.4 Fiscal Equalisation (FE)

As described in the main text (section 3.3), the fiscal equalisation scheme corresponds to an additional worldwide capital tax (t_f) whose revenues are redistributed among the countries according to their population shares. The net public consumption, denoted G_i instead of g_i, is then

$$G_i = \underbrace{t_i k_i}_{g_i} - t_f k_i + t_f \underbrace{(s_i k_i + s_j k_j)}_{k^*} = t_i k_i + t_f[k^* - k_i] = t_f k^* + [t_i - t_f]k_i,$$

but private consumption is unchanged, $x_i = f(k_i) + [k^* - k_i]f'(k_i) - t_1 k^*$ for given taxes. We have thus

$$\begin{aligned} \frac{\partial x_i}{\partial t_i} &= [k^* - k_i]f''(k_i)\frac{\partial k_i}{\partial t_i} - k^* \\ \frac{\partial G_i}{\partial t_i} &= k_i + [t_i - t_f]\frac{\partial k_i}{\partial t_i}. \end{aligned}$$

Proposition IV.1 (symmetric case)

The capital-based fiscal equalisation system raises the Nash equilibrium tax rates in a symmetric equilibrium (i.e., for $s_i = 0.5$). The equilibrium tax rates are then strictly increasing in the fiscal equalisation parameter t_f, and there exists a t_f which implements the first-best outcome.

Proof of Proposition IV.1:

Also in the presence of a FE Scheme, we have in the symmetric equilibrium $k_i = k^*$, yielding still $\partial k_i / \partial t_i = [2f''(k^*)]^{-1}$.

While without FE we had (tax rate for symmetric standard Nash equilibrium, indexed N) $FOC_N: \; U_g(k_i + t_i \cdot [2f''(k^*)]^{-1}) + U_x(-k^*) \stackrel{!}{=} 0$, and thus $t_N = -2k^* f''(k^*)[1 - \frac{U_x}{U_g}]$, we have now with FE (also for the symmetric situation $s_i = 0.5$, and indexed FE):

$$FOC_{FE}: \; U_g(k_i + [t_i - t_f] \cdot [2f''(k^*)]^{-1}) + U_x(-k^*) \stackrel{!}{=} 0,$$

leading to

$$t_{FE} = t_f - 2k^* f''(k^*)[1 - \frac{U_x}{U_g}]. \tag{A.51}$$

Noting that $\frac{U_x}{U_g}$ is increasing in the symmetric tax rate[11], the following simple comparative statics confirms that (A.51) indeed necessarily implies $t_{FE} > t_N$ as intuitive judgement would suggest when looking at (A.51):

Calling $a \equiv -2k^* f''(k^*)$ and noting $a > 0$, the two taxes write $t_N = a - a \left.\frac{U_x}{U_g}\right|_{t_N}$ and $t_{FE} = t_f + a - a \left.\frac{U_x}{U_g}\right|_{t_{FE}}$. Denoting $\left.\frac{U_x}{U_g}\right|_{t_{FE}} - \left.\frac{U_x}{U_g}\right|_{t_N} = c[t_{FE} - t_N]$, where we know that $c > 0$ (without knowing its magnitude), and subtracting the two tax rate expressions we have $[t_{FE} - t_N] = t_f - ac[t_{FE} - t_N]$, yielding $t_{FE} - t_N = \frac{t_f}{1+ac}$, where we know that the denominator on the RHS is positive, wherewith we know that in a symmetric Nash equilibrium, the capital tax under an FE scheme is indeed unambiguously larger than without FE. What's more, it is straightforward to see in our comparative statics that the equilibrium taxes are monotonously increasing in the fiscal equalisation parameter t_f. Finally, noting that the socially optimal tax rate is implicitly characterized by $u_x = u_g$, it is straightforward to see in (A.51) that a large enough t_f implements the first-best outcome. ■ *Proposition IV.1.*

Proposition IV.2 (asymmetric case)

[11] Increasing the tax in both countries by the same amount raises g_i and reduces x_i, lowering the relative marginal utility of the public good.

If both countries compete in a way such that each country's optimal reaction to a tax rate increase by the other country is an increase or no change of the domestic tax rate, then a limited fiscal equalisation scheme leads to an increase in the two countries' tax rates. In this case, the utility of the larger country necessarily increases. When the relative sizes of the countries are close enough to 0.5, the utility of the smaller country increases as well with the FE scheme. The right equalisation parameter t_f implements the first-best outcome.

Proof of Proposition IV.2:
We have now $\frac{\partial k_i}{\partial t_i} = \frac{1}{f''(k_i)+\frac{s_i}{1-s_1}f''(k_j)} = \frac{s_j}{s_j f''(k_i)+s_i f''(k_j)}$ (which yields at $t_i = t_j : \frac{\partial k_i}{\partial t_i} = \frac{1-s_i}{f''(k^*)}$) and (indexing expressions for country i under the FE scheme by i, FE)

$$FOC_{i,FE}: \quad U_g(k_i + [t_i - t_f] \cdot \frac{s_j}{s_j f''(k_i)+s_i f''(k_j)}) + U_x([k^* - k_i] f''(k_i) \frac{s_j}{s_j f''(k_i)+s_i f''(k_j)} - k^*) \stackrel{!}{=} 0,$$

leading to

$$t_i = t_f + \underbrace{\frac{U_x}{U_g}(k_i f''(k_i) + k^* \frac{s_i f''(k_j)}{s_j})}_{<0} - k_i \frac{s_j f''(k_i) + s_i f''(k_j)}{s_j} \; \forall i,j | i \neq j, \qquad (A.52)$$

where we note that this expression is exactly the same as that for the case without FE, except the t_f at the beginning on the RHS, which would not be there.

The analysis requires now consideration of two equations. Consider an initial 'starting' (index s) equilibrium with optimal taxes t_i^s which must respect eqs. (A.52) for $t_f = 0$. For the taxes to represent an equilibrium we know that $t_i^s = t_i^R(t_j^s)$ hold, where $t_i^R(t_j^s)$ are the tax reaction functions for the case without fiscal equalization.

Defining a_i the first term after the t_f on the RHS of (A.52), $a_i \equiv \frac{U_x}{U_g}(k_i f''(k_i) + k^* \frac{s_i f''(k_j)}{s_j})$ and b_i the second (including the leading minus sign),
$b_i \equiv -k_i \frac{s_j f''(k_i)+s_i f''(k_j)}{s_j}$, and noting that k_i (and also its derivatives with respect to taxes) only depend on the difference between the taxes, and $f(\cdot)$ and thus also b_i is purely a function of k, we rewrite (A.52) as

$$t_1 = t_f + a_1(t_1, t_2) + b_1(t_1 - t_2) \qquad (A.53)$$
$$t_2 = t_f + a_2(t_2, t_1) + b_2(t_1 - t_2). \qquad (A.54)$$

Denoting $a'_{i,1}$ and $a'_{i,2}$ the derivative of a_i with respect to its first resp. its second argument, we can solve the system explicitly for marginal deviations of t_f from zero, starting from the initial equilibrium of t_i^s (where $t_f = 0$), by first order approximation, denoting $\Delta_i = t_i - t_i^s$ the deviations of the new equilibrium taxes t_i from the initial equilibrium:

$$\Delta_1 = \frac{t_f}{1 - a'_{1,1} - b'_1} + \Delta_2 \frac{a'_{1,2} - b'_1}{1 - a'_{1,1} - b'_1} \qquad (A.55)$$
$$\Delta_2 = \frac{t_f}{1 - a'_{2,1} + b'_2} + \Delta_1 \frac{a'_{2,2} + b'_1}{1 - a'_{2,1} + b'_2}. \qquad (A.56)$$

Recalling Lemma 3, $\frac{\partial t_i^R(t_j)}{\partial t_j} < 1$, we know that in an equilibrium, a marginal deviation of the foreign tax rate increases the domestic tax rate by less than 1. Assuming in addition that starting from an equilibrium, the optimal response of the domestic tax rate to an increase in the foreign tax rate is never negative, we know that the second fraction on the RHS of either of eq. (A.55) and (A.56) is larger or equal to zero and lower than 1. Defining $\alpha_1 \equiv a'_{1,2} - b'_1$, $\beta_1 \equiv \left[1 - a'_{1,1} - b'_1\right]^{-1}$, $\alpha_2 \equiv a'_{2,2} + b'_1$ and $\beta_2 \equiv \left[1 - a'_{2,1} + b'_2\right]^{-1}$ we thus have

$$0 \leq \alpha_i \beta_i < 1. \tag{A.57}$$

In addition, in order for the system to be stable, that is, in order for in the initial state the tax t_i not to unilaterally diverge from its equilibrium value t_i^s, it is easy to see in eqs. (A.53) and (A.54) that we must have $-1 < a'_{1,1} + b'_1 < 1$ and $-1 < a'_{2,1} - b'_2 < 1$, which then also implies $\beta_i > 0$ and thus, because of (A.57), also $\alpha_i > 0$.

Solving the system $\Delta_i = t_f \beta_i + \Delta_j \alpha_i \beta_i \, \forall i \neq j$ explicitly for the two tax rates yields

$$\Delta_i = t_f \frac{\beta_i + \beta_j \alpha_i \beta_i}{1 - \alpha_j \beta_j \alpha_i \beta_i} \, \forall i \neq j,$$

in which we know that all variables are positive, as well as that $\alpha_i \beta_i < 1 \, \forall i$, implying

$$\frac{t_i|_{t_f} - t_i^s}{t_f} > 0,$$

which is valid at least for marginal t_f.

It is straightforward to see that the utility of the larger country, which has the higher tax (Proposition I.2), increases with the FE: Assume country 1 is the large country and country 2 the small. Consider the initial situation without FE, where the taxes are t_i^s, implying a specific utility u_1^s. Now, consider a change of country 2's tax to the higher level in the ultimately reached FE equilibrium, but hypothetically consider this move to t_2 to be unilateral, without any FE being introduced. Because $t_2 > t_2^s$, increasing the capital intensity in country 2, we obviously have $U_1|_{t_1^s,t_2}^{no\,FE} > U_1|_{t_1^s,t_2^s}^{no\,FE}$. Second, assume now that the FE scheme is introduced after that first move to t_2, but first without any subsequent tax changes. Because the FE scheme redistributes public good from country 2 to country 1 (recall that country 2 is the small country and its tax remains lower even after a marginal change), we obviously have $U_1|_{t_1^s,t_2}^{FE} > U_1|_{t_1^s,t_2}^{no\,FE}$. Finally, because country voluntarily chooses to move from this state to one with a different tax $t_1 \neq t_1^s$, this must increase its utility, thus $U_1|_{t_1,t_2}^{FE} > U_1|_{t_1^s,t_2}^{FE}$. Strictly speaking, while the conclusion is right, the last sentence needs some qualification: there would probably be no equilibrium with the considered FE scheme and taxes (t_1, t_2^s), as country 2 would generally choose a different tax rate than t_2^s if country 1 differs from its equilibrium-counterpart t_1^s. Because t_1^s is, however, an *unstrategic* Nash equilibrium choice (as opposed to, e.g., a strategic Stackelberg-leader choice), we still know that country 1 must be better off with (t_1^s, t_2^s) under the FE scheme than in the hypothetical situation (t_1, t_2^s) with the same FE, as even if country 2's tax might react on a deviation of country 1's tax from t_1^s, country 1 would not account for that and deviate from t_1^s regardless of any potential losses due to subsequent moves from country 2. This behaviour is the direct consequence of the concept of the Nash equilibrium, which is an equilibrium only if no party has any incentive to unilaterally deviate regardless of the incentives an unilateral deviation may create for the reaction of the other parties. The three found inequalities for the utilities establish that we must have $U_1|_{t_1,t_2}^{FE} > U_1|_{t_1^s,t_2^s}^{no\,FE}$.

Further see: When the sizes of the countries are close enough, the utility of the smaller country also increases with t_f. We see this by an extension of the proof for Proposition IV.1, adapted for country sizes that do not exactly but only nearly match each other:

Subproof: for relative country sizes approaching 0.5, an FE scheme increases the utility of both countries.

We assume well-behaved underlying functions. As the sizes of the two countries approach each other, $\lim s_i \to 0.5$, we have $\lim_{s_i \to 0.5} k_i = k^*$, yielding further $\lim_{s_i \to 0.5} \partial k_i / \partial t_i = [2f''(k^*)]^{-1}$. Thus, without FE we would in this case have (indexing expressions for country i under the standard Nash competition without FE by i, N) $FOC_{i,N}$: $\lim_{s_i \to 0.5} U_g(k_i + t_i \cdot [2f''(k^*)]^{-1}) + U_x(-k^*) \stackrel{!}{=} 0$, and thus $\lim_{s_i \to 0.5} t_{i,N} = -2k^* f''(k^*)[1 - \frac{U_x}{U_g}]$. With FE we have (also for $\lim s_i \to 0.5$):

$$FOC_{i,FE} : \lim_{s_i \to 0.5} U_g(k_i + [t_i - t_f] \cdot [2f''(k^*)]^{-1}) + U_x(-k^*) \stackrel{!}{=} 0,$$

leading to

$$\lim_{s_i \to 0.5} t_{i,FE} = t_f - 2k^* f''(k^*)[1 - \frac{U_x}{U_g}]. \tag{A.58}$$

Noting that $\frac{U_x}{U_g}$ is increasing if both countries increase their taxes simultaneously by the same amount in the quasi-symmetric case,[12] the following simple comparative statics confirms that (A.58) indeed necessarily implies $\lim_{s_i \to 0.5} t_{i,FE} > t_{i,N}$, as intuitive judgement would suggest when looking at (A.58):

Calling $a \equiv -2k^* f''(k^*)$ and noting $a > 0$, the limits of the two taxes write $\lim_{s_i \to 0.5} t_{i,N} = \lim_{s_i \to 0.5} a - a \left. \frac{U_x}{U_g} \right|_{t_{i,N}}$ and $\lim_{s_i \to 0.5} t_{i,FE} = \lim_{s_i \to 0.5} t_f + a - a \left. \frac{U_x}{U_g} \right|_{t_{i,FE}}$. Denoting $\left. \frac{U_x}{U_g} \right|_{t_{i,FE}} - \left. \frac{U_x}{U_g} \right|_{t_{i,N}} = c[t_{i,FE} - t_{i,N}]$, where we know that, at least for $\lim s_i \to 0$, we have $c > 0$ (we ignore its magnitude, however), and subtracting the two tax rate expressions we find $\lim_{s_i \to 0.5} [t_{i,FE} - t_{i,N}] = \lim_{s_i \to 0.5} t_f - ac[t_{i,FE} - t_{i,N}]$, yielding $\lim_{s_i \to 0.5} t_{i,FE} - t_{i,N} = \frac{t_f}{1+ac}$, where the denominator on the RHS is finite and positive. Thus, in a quasi-symmetric Nash equilibrium with $\lim s_i \to 0.5$, the capital taxes under an FE scheme must indeed be larger than without FE. Furthermore, because the capital intensities differ only marginally, the FE scheme induces on one hand only a marginal net transfer between the regions , but on the other hand it implies a tax increase in both countries that is (per unit of t_f) more than marginal. While a simultaneous (more than marginal) tax increase starting from the competitive quasi-symmetric case increases both countries' utilities by a more than marginal value, the marginal FE transfer can reduce the utility of a country by maximally a marginal amount. Overall, the changes induced by the FE thus on net necessarily increases the utilities of both countries. ■ *Subproof.*

Even in the asymmetric case for any possible s_i, the FE level can be set to a level such that the first-best outcome is achieved:

Note that in the first best outcome, both countries set the same tax rate, $t_i = t^*$, and have the same capital intensities $k_i = k^*$, and the marginal value of the public good is 1 in both countries, $\text{MRS}_i = 1$. In (A.52) it is easy to see that there exists a t_f^* for which the first-best outcome can be sustained: if we have $t_f = t^*$ and both countries choose $t_i = t_f = t^*$, then the sum of all values on the RHS of (A.52) except t_f collapses to zero, such that the equation is fulfilled for the first-best tax rates. Thus, even if $s_i \neq 0.5$, for $t_f = t^*$ the first best outcome, $t_i = t^*$, is achieved. ■ *Proposition IV.2.*

[12]Increasing the tax in both countries by the same amount raises g_i and reduces x_i, lowering the relative marginal utility of the public good.